 Nielsen Theory and Related Topics

Abstracts

July 4 – July 9, 2016

Department of Mathematics – IGCE

UNESP – São Paulo State University

Rio Claro – SP, Brazil
# Program

## Monday

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
<th>Speaker/Author</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>09:00 – 09:50</td>
<td>registration &amp; opening ceremony</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10:00 – 10:50</td>
<td>Robert F. Brown</td>
<td>pg. 19</td>
<td></td>
</tr>
<tr>
<td>10:50 – 11:20</td>
<td>coffee break</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11:20 – 12:10</td>
<td>Daciberg L. Gonçalves</td>
<td>pg. 04</td>
<td></td>
</tr>
</tbody>
</table>

### Lunch

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
<th>Speaker/Author</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>14:30 – 15:20</td>
<td>Xuezhi Zhao</td>
<td>pg. 28</td>
<td></td>
</tr>
<tr>
<td>15:30 – 16:20</td>
<td>Ulrich Koschorke</td>
<td>pg. 23</td>
<td></td>
</tr>
<tr>
<td>16:25 – 16:55</td>
<td>coffee break</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17:00 – 17:25</td>
<td>Gert-Jan Dugardein</td>
<td>pg. 11</td>
<td></td>
</tr>
<tr>
<td>17:30 – 17:55</td>
<td>Gustavo de L. Prado</td>
<td>pg. 12</td>
<td></td>
</tr>
</tbody>
</table>

18:30            | cocktail                                      |                |      |

## Tuesday

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
<th>Speaker/Author</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>09:00 – 09:50</td>
<td>Chris Staecker</td>
<td>pg. 02</td>
<td></td>
</tr>
<tr>
<td>10:00 – 10:25</td>
<td>Dirceu Penteado</td>
<td>pg. 08</td>
<td></td>
</tr>
<tr>
<td>10:30 – 11:00</td>
<td>coffee break</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11:00 – 11:50</td>
<td>Alexander Felsh'tyn</td>
<td>pg. 01</td>
<td></td>
</tr>
</tbody>
</table>

### Lunch

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
<th>Speaker/Author</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>14:30 – 15:20</td>
<td>Timur Nasybullov</td>
<td>pg. 22</td>
<td></td>
</tr>
<tr>
<td>15:30 – 16:20</td>
<td>Karel Dekimpe</td>
<td>pg. 14</td>
<td></td>
</tr>
<tr>
<td>16:25 – 16:55</td>
<td>coffee break</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17:00 – 17:25</td>
<td>Taciana O. Souza</td>
<td>pg. 20</td>
<td></td>
</tr>
<tr>
<td>17:30 – 17:55</td>
<td>Daniel Gottlieb</td>
<td>pg. 05</td>
<td></td>
</tr>
</tbody>
</table>

18:30            | free & social activities                      |                |      |

## Wednesday

<table>
<thead>
<tr>
<th>Time</th>
<th>Activity</th>
<th>Speaker/Author</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>09:30 – 10:20</td>
<td>Waclaw Marzantowicz</td>
<td>pg. 25</td>
<td></td>
</tr>
<tr>
<td>10:20 – 11:10</td>
<td>J.B. Lee</td>
<td>pg. 13</td>
<td></td>
</tr>
<tr>
<td>11:20 – 12:20</td>
<td>Posters &amp; coffee break</td>
<td>pg. 29</td>
<td></td>
</tr>
</tbody>
</table>

### free & social activities

---
Thursday

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>09:00 – 09:50</td>
<td>Daniel Vendruscolo</td>
<td>06</td>
</tr>
<tr>
<td>10:00 – 10:25</td>
<td>Vinicius C. Laass</td>
<td>24</td>
</tr>
<tr>
<td>10:30 – 11:00</td>
<td>coffee break</td>
<td></td>
</tr>
<tr>
<td>11:00 – 11:50</td>
<td>Michael Kelly</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>lunch</td>
<td></td>
</tr>
<tr>
<td>14:30 – 15:20</td>
<td>Thaís F.M. Monis</td>
<td>21</td>
</tr>
<tr>
<td>15:30 – 16:20</td>
<td>Edivaldo L. dos Santos</td>
<td>10</td>
</tr>
<tr>
<td>16:25 – 16:55</td>
<td>coffee break</td>
<td></td>
</tr>
<tr>
<td>17:00 – 17:25</td>
<td>Claudemir Aniz</td>
<td>03</td>
</tr>
<tr>
<td>17:30 – 17:55</td>
<td>Northon C.L. Penteado</td>
<td>16</td>
</tr>
<tr>
<td>18:30</td>
<td>dinner</td>
<td></td>
</tr>
</tbody>
</table>

Friday

<table>
<thead>
<tr>
<th>Time</th>
<th>Speaker</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>09:30 – 10:20</td>
<td>Philip Heath</td>
<td>18</td>
</tr>
<tr>
<td>10:20 – 10:50</td>
<td>coffee break</td>
<td></td>
</tr>
<tr>
<td>11:00 – 11:50</td>
<td>Weslem L. Silva</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>lunch</td>
<td></td>
</tr>
<tr>
<td>14:30 – 15:20</td>
<td>Davide Ferrario</td>
<td>07</td>
</tr>
<tr>
<td>15:30 – 16:20</td>
<td>Peter Wong</td>
<td>17</td>
</tr>
<tr>
<td>16:25 – 16:55</td>
<td>coffee break</td>
<td></td>
</tr>
<tr>
<td>17:00 – 18:00</td>
<td>problems session</td>
<td></td>
</tr>
<tr>
<td>18:30</td>
<td>free &amp; social activities</td>
<td></td>
</tr>
</tbody>
</table>

Download this file here
Talks

Nielsen–Reidemeister theory, dynamical zeta functions and topological entropy on infra-solvmanifolds of type $R$ ................................................................. 1
Alexander Felshtyn and Jong Bum Lee

The expectation of the error between the Nielsen number and the minimum number of fixed points ................................................................. 2
Chris Staecker and S.W. Kim

Linear systems over $\mathbb{Z}[Q_{16}]$ and roots of maps of some 3-complexes into $M_{Q_{16}}$ .......... 3
Claudemir Aniz

The fixed points of multimaps on surface with application to the torus – a Braid approach 4
Daciberg Lima Gonçalves and John Guaschi

On space-like vector fields and time-like vector fields on Minkowski space time and their relationship with light-like vector fields and null vector fields ......................... 5
Daniel Gottlieb

Nielsen Borsuk–Ulam coincidence theory on surfaces ........................................ 6
John Guaschi and Daniel Vendrúscolo

Central configurations and equivariant fixed points ........................................ 7
D.L. Ferrario

Fibre maps and fixed points on certain Surface Bundles ........................................ 8
D. L. Gonçalves, A. K. M. Libardi, D. Penteado and J. P. Vieira

Zero sets of equivariant maps from products of spheres to euclidean spaces .......... 10
Denise de Mattos, Pedro Luiz Queiroz Pergher, Edivaldo Lopes dos Santos and Mahender Singh

A covering space approach to fixed point classes on n-valued multimaps ............... 11
Gert-Jan Dugardein

Wecken type results in codimension one .......................................................... 12
Gustavo de Lima Prado

Growth rate for endomorphisms of finitely generated nilpotent groups ................ 13
A. Fel’shtyn, J.H. Jo and J.B. Lee

Recent developments on the $R_\infty$ property for nilpotent quotients of groups .......... 14
Karel Dekimpe and Daciberg L. Gonçalves

Index bounds for complexes associated to limit groups .................................... 15
Daciberg L. Gonçalves and Michael R. Kelly

Representing Homotopy Classes by Special Maps ........................................... 16
Oziride Manzoli Neto and Northon Canevari Leme Penteado
Obstruction theory for coincidences of multiple maps .......................... 17
Thais Monis and Peter Wong

Nielsen numbers of iterates and Nielsen type periodic numbers of periodic maps on tori . 18
Philip R. Heath

Nielsen numbers of n-valued maps ...................................................... 19
Robert F. Brown

(H, G)-Coincidence theorems and a Topological Tverberg type theorem for manifolds .... 20
Denise de Mattos, Edivaldo Lopes dos Santos and Taciana O. Souza

Remarks on the fixed point property for flag manifolds .............................. 21
Thais F.M. Monis

Properties of twisted conjugacy classes in some groups ............................ 22
Timur Nasybullov

Coincidences and secondary Nielsen numbers ........................................ 23
Ulrich Koschorke

The Borsuk–Ulam problem for homotopy class of functions. An approach using braid groups ................................................................. 24
Vinicius Casteluber Laass

Topological complexity of spaces with the presence of actions of finite groups ........ 25
Wacław Marzantowicz

Periodic points on Torus fiber bundles over the circle ................................ 27
Weslem Liberato Silva and Rafael Moreira de Souza

Common value classes ................................................................. 28
Xuezhi Zhao
NIELSEN–REIDEMEISTER THEORY, DYNAMICAL ZETA FUNCTIONS AND TOPOLOGICAL ENTROPY ON INFRA-SOLVMANIFOLDS OF TYPE $R$

Alexander Fel’shtyn$^1$ and Jong Bum Lee$^2$

Abstract

We prove the rationality, the functional equations and calculate the radii of convergence of the Nielsen, the Reidemeister and the Artin-Mazur zeta functions of continuous maps on infra-solvmanifolds of type $R$. We find a connection between the topological entropy, the Reidemeister and Nielsen zeta functions and the Reidemeister torsions of the corresponding mapping tori. We show that if the Reidemeister zeta function is defined for a homeomorphism on an infra-solvmanifold of type $R$, then this manifold is an infra-nilmanifold. We also prove that a map on an infra-solvmanifold of type $R$ induced by an affine map minimizes the topological entropy in its homotopy class. We prove the Gauss congruences for the Reidemeister and Nielsen numbers of any map on an infra-solvmanifolds of type $R$ whenever all the Reidemeister numbers of iterates of the map are finite. We study the asymptotic behavior of the sequence of the Nielsen numbers of iterations, the essential periodic orbits of a map and the homotopy minimal periods of a map by using the Nielsen theory of maps on infra-solvmanifolds of type $R$. We give a linear lower bound for the number of essential periodic orbits of such a map, which sharpens known results for periodic points and for periodic orbits. We also verify that a constant multiple of infinitely many prime numbers occur as homotopy minimal periods of such a map.

References


$^1$University of Szczecin, Poland. felshtyn@gmail.com

$^2$Sogang University, Seoul, Korea. jlee@sogang.ac.kr
LINEAR SYSTEMS OVER $\mathbb{Z}[Q_{16}]$ AND ROOTS OF MAPS OF SOME 3-COMPLEXES INTO $M_{Q_{16}}$

Claudemir Aniz$^1$

Abstract

Let $\mathbb{Z}[Q_{16}]$ be the group ring where $Q_{16}$ is the quaternion group of order 16 and $\varepsilon$ the augmentation map. We show that, if $PX = K(x - 1)$ and $PX = K(-xy + 1)$ has solution over $\mathbb{Z}[Q_{16}]$ and all $m \times m$ minors of $\varepsilon(P)$ are relatively prime, then the linear system $PX = K$ has solution over $\mathbb{Z}[Q_{16}]$, where $P = [p_{ij}]$ is an $m \times n$ matrix with $m \leq n$. As a consequence of such results, we show that there is no map $f : W \to M_{Q_{16}}$ that is strongly surjective, i.e., such that $\text{MR}[f,a] = \min\{\#(g^{-1}(a)) | g \in [f]\} \neq 0$. Here, $M_{Q_{16}}$ is the orbit space of the 3-sphere $S^3$ with respect to the action of the $Q_{16}$ determined by the inclusion $Q_{16} \subseteq S^3$ and $W$ is a CW-complex of dimension 3 with $H^3(W;\mathbb{Z}) = 0$.

References


$^1$Federal University of Mato Grosso do Sul. claudemir.aniz@ufms.br
THE EXPECTATION OF THE ERROR BETWEEN THE NIELSEN NUMBER AND THE MINIMUM NUMBER OF FIXED POINTS

Chris Staecker and S.W. Kim

Abstract

Recent work has shown that when $X$ is a surface with boundary having fundamental group of rank $n$ and $f$ is chosen at random, then $N(f) = MF(f)$ with probability approaching 1 as $n \to \infty$.

This is in spite of an infinite family of examples due to Kelly which show that $N(f)$ can be arbitrarily smaller than $MF(f)$ for certain $f$. The present paper measures the expectation of the quantity $MF(f) - N(f)$ when $f$ is chosen at random. We show that this expectation approaches 0 as $n \to \infty$, which indicates that Kelly’s examples are exceptional, rather than typical. For individual $n$, the expectation of $MF(f) - N(f)$ may be nonzero, and we give a lower bound in the case $n = 2$. We also consider the expectation of $MF(f)/N(f)$, and show that this expectation is 1 for each $n$. 

1
The fixed points of multimaps on surface with application to the torus- a Braid approach

Daciberg Lima Gonçalves¹ and John Guaschi

Abstract

Let \( \phi : S \to S \) be a \( n \)-valued continuous multimap on some compact surface \( S \). First we classify the homotopy classes of multimaps where for most of the surfaces the classification is given in terms of the braids on \( n \)-strings of the surface \( S \). Then we give an algebraic criterion to decide which homotopy classes contain a multimap which is fixed point free. We will focus on the cases where \( S \) is a closed surface of Euler characteristic \( \leq 0 \). Despite the fact that the algebraic criterion is quite hard, we perform some specific calculations for the case where \( S \) is the torus. The concept of Nielsen number for a surface has been developed. Then I explain the status of the Wecken property for multimaps on the torus. In fact it is an open question if there is an example of a multimap which has Nielsen number zero but it can not be deformed to fixed point free. Finally a brief exposition about the case of the projective plane should be presented. Below are some of the relevant references for our purpose.

References


¹Department of Mathematics-University of São Paulo.
digoncal@ime.usp.br
ON SPACE-LIKE VECTOR FIELDS AND TIME-LIKE VECTOR FIELDS ON MINKOWSKI SPACE TIME AND THEIR RELATIONSHIP WITH LIGHT-LIKE VECTOR FIELDS AND NULL VECTOR FIELDS

Daniel Gottlieb\textsuperscript{1}

Abstract

The hyperbolic sine and cosine plays an important role, showing up in Lorentz Transformations, and in Maxwell’s equations.

\textsuperscript{1}USA daniel.gottlieb@gmail.com
Nielsen Borsuk–Ulam coincidence theory on surfaces

John Guaschi\textsuperscript{1} and Daniel Vendrúscolo\textsuperscript{2}

Abstract

In this work we discuss the existence and the minimal number of Borsuk-Ulam coincidence points for homotopy classes of maps from a surface with a free involution to another surface.

Given two topological spaces \(X\) and \(Y\) such that \(X\) admits a free involution \(\tau\), we say the triple \((X, \tau, Y)\) satisfy the Borsuk-Ulam Property if for any continuous \(f : X \to Y\) there exist a point \(x \in X\) such that \(f(x) = f(\tau(x))\).

A point such that \(f(x) = f(\tau(x))\) is a Borsuk-Ulam coincidence. Recently the question about the existence of Borsuk-Ulam coincidence points has been studied for homotopy classes of maps, in a "Nielsen Theory" approach. In the same way as the usual Nielsen coincidence theory the case of dimension 2 is more difficult.

Using braids we describe the minimal number of Borsuk-Ulam coincidence points for homotopy classes of maps \(f : M \to N\) between surfaces (with \(\pi_2(N) = 0\)).

\textsuperscript{1}Normandie Université, UNICAEN, Laboratoire de Mathématiques Nicolas Oresme UMR CNRS 6139, 14032 Caen Cedex, France john.guaschi@unicaen.fr
\textsuperscript{2}Departamento de Matemática - UFSCar daniel@dm.ufscar.br
CENTRAL CONFIGURATIONS AND EQUIVARIANT FIXED POINTS

D.L. Ferrario

Abstract

Central configurations of \( n \) point particles in a euclidean space interacting via a potential function \( U \) are shown to be the same as the fixed points of the normalized gradient map, which is an \( SO(d) \)-equivariant self-map defined on the inertia ellipsoid. We then show how to relate Morse-indices of central configurations (as critical points) with their fixed point indices. The approach is then used to give some estimates of the number of central configurations and to relate bifurcations of the energy surfaces in the configuration spaces with Nielsen fixed point classes.

References

Abstract

Given a fibration \( E \to B \) and \( h : E \to E \) a fibre-preserving map over \( B \), the question if \( h \) can be deformed over \( B \) (by a fibrewise homotopy) to a fixed point free map has been considered for several years by many authors. Among others, see for example [Dol74], [FH81], [Gon87], [Pen97], [GPV04], [GPV09I], [GPV09II] and [GLPV13]. More recently also the fibrewise coincidence case has been considered in [Kos11], [GK09], [GPV10], [SV12], [Vie] and [GKLN], which certainly has intersection with the fixed point case.

The present work is a continuation of the work [GLPV13] and the main stream is the study of fixed point theory of surface bundles. More details about the development and the state of art of this case one can find in [GLPV13]. There, the problem has been solved in the case where the surface bundle is the trivial bundle \( S^1 \times S^2 \) where \( S^2 \) is the closed orientable surface of genus 2. Here we extend the work [GLPV13] where we considered the case of surface trivial bundle \( B \times M \) over an arbitrary base \( B \) (not necessarily \( S^1 \)) and the fibre \( M \) is a closed surface of negative Euler characteristic. So \( M \) is either \( S_g \) or \( N_g \), where \( S_g \) is the closed orientable surface of genus \( g > 1 \) and \( N_g \) is the closed non-orientable surface of genus \( g > 2 \) (i.e. the sum of \( g + 1 \) projective planes). More precisely we study fibrewise maps of the trivial bundles \( B \times S_g \) and \( B \times N_g \).

The main purpose of this work is to study fixed points of fibre-preserving maps over \( B \) on the trivial surface bundles \( B \times S \) where \( S \) is a closed surface of negative Euler characteristic. The case where \( B = S^1 \) and \( S \) is equal to \( S_2 \), i.e., the closed orientable surface of genus 2, is already known. We classify all such maps that can be deformed fibrewise to a fixed point free map.

The main result of this paper is:

**Theorem.** A fibrewise map \( h \) can be deformed over \( B \) to a fixed point free map if and only if \( h \) is fibrewise homotopic to \( id \times g \) where \( g : S \to S \) is a fixed point free map homotopic to \( f \) restricted to \( x_0 \times S \).

References


---

This work was partially supported by Fapesp


Zero sets of equivariant maps from products of spheres to euclidean spaces

Denise de Mattos$^1$ and Pedro L. Q. Pergher$^2$ and
Edivaldo L. dos Santos$^3$ and Mahender Singh$^4$

Abstract

Let $E \to B$ be a fiber bundle and $E' \to B$ be a vector bundle. Let $G$ be a compact group acting fiber preservingly and freely on both $E$ and $E' - 0$, where $0$ is the zero section of $E' \to B$. Let $f : E \to E'$ be a fiber preserving $G$-equivariant map, and let $Z_f = \{x \in E \mid f(x) = 0\}$ be the zero set of $f$. In [2], Albrecht Dold obtained a lower bound for the cohomological dimension of the zero set $Z_f$ when $E \to B$ is the sphere bundle associated to a vector bundle which is equipped with the antipodal action of $\mathbb{Z}/2$. In [3], Denise de Mattos and Edivaldo L. dos Santos extended this Dold result considering bundles whose fiber is a product of two spheres. In [1], D. M. Davis determined the structure of the cohomology ring of the orbit space of an arbitrary product of spheres by the diagonal antipodal action of $\mathbb{Z}/2$. With this additional tool in hand, recently we extended the above results for bundles whose fiber is an arbitrary product of spheres, equipped with the mentioned action. We also proved a Bourgin-Yang type theorem for products of spheres equipped with the diagonal antipodal action of $\mathbb{Z}/2$. These results are the content of a recent paper, accepted for publication at December-2015, and published in "Topology and its Applications", as a joint work of Denise de Mattos, Edivaldo L. dos Santos, Mahender Singh and Pedro L. Q. Pergher. This lecture will focus the content in question.

References


$^1$Departamento de Matematica, Instituto de Ciencias Matematicas e Computacao, Universidade de Sao Paulo. denisem@icmc.usp.br
$^2$Departamento de Matematica, Universidade Federal de Sao Carlos, Centro de Ciencias Exatas e Tecnologia. pergher@dm.ufscar.br
$^3$Departamento de Matematica, Universidade Federal de Sao Carlos, Centro de Ciencias Exatas e Tecnologia. edivaldo@dm.ufscar.br
$^4$Indian Institute of Science Education and Research (IISER), mahender@iisermohali.ac.in
A COVERING SPACE APPROACH TO FIXED POINT CLASSES ON $n$-VALUED MULTIMAPS

Gert-Jan Dugardein$^1$

Abstract

In this talk, I will present a method to define fixed point classes on $n$-valued multimaps by using a covering space approach. By doing so, it is possible to define a Reidemeister number for these multivalued maps. I will show that this approach agrees with the work of Schirmer and I will demonstrate the advantages by applying this new method to some maps on infra-nilmanifolds.

$^1$KU Leuven Kulak, E. Sabbelaan 53, B-8500 Kortrijk
gertjan.dugardein@kuleuven-kulak.be


Wecken type results in codimension one

Gustavo de Lima Prado

Abstract

Let \( f_1, f_2 : X^{n+1} \rightarrow Y^n \) be maps, where \( X, Y \) are smooth manifolds, connected, being \( X \) closed (compact and without boundary) and \( Y \) without boundary. A Wecken type result is a result which asserts that, under some conditions, the minimum number of components of the coincidence set is equal to the Nielsen number, that is, \( MCC(f_1, f_2) = \tilde{N}(f_1, f_2) \) for all pairs of maps. In [2], U. Koschorke obtains Wecken type results for \( n = 1 \) and for the stable dimension (which, in the codimension one case, means \( n \geq 4 \)). In this work, when \( X = S^{n+1} \), we obtain a Wecken type result for \( n = 2, 3 \). Hence, in the codimension one case, when the domain is the sphere, there exists a Wecken type result for every dimension. Also we obtain a Wecken type result for \( X = \mathbb{R}P(3) \) and \( Y = S^2 \). This work is part of the author’s thesis, [3], under the advisory of Daciberg Lima Gonçalves and Ulrich Koschorke.

References


---

1Federal University of Uberlândia. glprado@ufu.br
Abstract

Let $\pi$ be a finitely generated group with a system $S = \{s_1, \ldots, s_n\}$ of generators. Let $\varphi: \pi \to \pi$ be an endomorphism. For any $\gamma \in \pi$, let $L(\gamma, S)$ be the length of the shortest word in the letters $S \cup S^{-1}$ which represents $\gamma$. Then the growth rate of $\varphi$ is defined to be

$$\text{GR}(\varphi) := \sup \left\{ \limsup_{k \to \infty} \frac{L(\varphi^k(\gamma), S)}{k} \mid \gamma \in \pi \right\}.$$ 

The algebraic entropy of $\varphi$ is by definition $\log \text{GR}(\varphi)$.

We prove that the growth rate of an endomorphism of a finitely generated nilpotent group equals to the growth rate of the induced endomorphism on its abelinization, generalizing the corresponding result for an automorphism in [2].

References


Recent developments on the $R_\infty$ property for nilpotent quotients of groups

Karel Dekimpe\textsuperscript{1} and Daciberg L. Gonçalves\textsuperscript{2}

Abstract

Any endomorphism $\varphi$ of a group $G$ determines an equivalence relation on $G$ by setting $x \sim y \iff \exists z \in G : x = zy\varphi(z)^{-1}$. The equivalence classes of this relation are called Reidemeister classes or twisted conjugacy classes and their number is denoted by $R(\varphi)$. These Reidemeister classes are also connected to the study of fixed points, where they can be interpreted as fixed point classes. There has been a growing interest in the groups $G$ having the $R_\infty$-property, these are groups for which $R(\varphi) = \infty$ for all automorphisms $\varphi$ of $G$.

Given a group $G$ having the $R_\infty$-property, we define the the $R_\infty$-nilpotency degree of $G$ as the least least integer $c$ such that the group $G/\gamma_{c+1}(G)$ has the $R_\infty$ property, if such a $c$ exists. Here $\gamma_i(G)$ stands for the $i$-th term of the lower central series of $G$. In this talk we restate the results we obtained earlier in case $G$ is a free group or a surface group in terms of this $R_\infty$-nilpotency degree and we also discuss our recent results in case $G$ is a Baumslag–Solitar group.

\textsuperscript{1}\textit{KU Leuven (Campus Kulak Kortrijk)} Karel.Dekimpe@kuleuven.be

\textsuperscript{2}\textit{USP. dlgoncal@ime.usp.br}
INDEX BOUNDS FOR COMPLEXES ASSOCIATED TO LIMIT GROUPS

Daciberg L. Gonçalves¹ and Michael Kelly²

Abstract

In the papers [1] and [2] the authors independently establish bounds for the indices of the Nielsen fixed point classes for self-maps of compact surfaces. Assuming the Euler characteristic $\chi$ of the surface is non-positive one obtains an upper bound of +1 and a lower bound given by $\sum (\text{index}(C) + 1) \geq 2\chi$, where the summation is over fixed point classes $C$ which have negative index.

The purpose of this research is to find a broad class of complexes for which these same bounds hold. We consider a family which arise in the study of equations on free groups. In particular, in this talk we consider the base case of a general construction, namely a 2-complex obtained by the identification of two surfaces along a simple closed curve in each of the surfaces. Given any self-map of one such complex we establish the same bounds as in the surface case. The proof in this case takes advantage of refined bounds in the surface case given in [3]. Also generalizations of classical surface topology results due to Hopf, Kneser and others.

References


¹USP–São Paulo. dlgoncal@ime.usp.br
²Loyola University–New Orleans kelly@loyno.edu
Representing Homotopy Classes by Special Maps

Oziride Manzoli Neto1 and Northon Canevari Leme Penteado2

Abstract

Given two topological spaces $X$ and $Y$ and a map $f : X \to Y$. The set $f^{-1}(y)$ is called root set of $f$ in $y$, for each $y \in Y$.

We are interested to determine, up to homotopy, the minimal root set of $f$.

Let $[f]$ a free homotopy class of a map $f : S^2 \to S^2 \vee S^1$. In the work [3] was made a characterization, up to homotopy, of the minimal root sets. Furthermore, it present the maps that realize this minimal root sets.

The [4] works in this problem and generalize [3], taking a closed, connected and compact surface as domain of maps.

The objective of this work is to present this characterization and will show an approach when the codomain of maps are obtained by the union by a point of one $S^1$ and $n$ spheres $S^2$. Moreover, we will show some problems when we get certain spaces as codomain of maps.

References

[4] Leme Penteado, N. C., Manzoli Neto, O., Raízes de aplicações de superfícies em $S^2 \vee ... \vee S^2 \vee S^1$, Universidade de São Paulo, PhD thesis, 2015.

1Universidade de São Paulo - São Carlos. ozimneto@icmc.usp.br
2Universidade Estadual Paulista - Rio Claro. northoncanevari@gmail.com
**Nielsen numbers of iterates and Nielsen type periodic numbers of periodic maps on tori**

Philip R. Heath

**Abstract**

Since for self maps of tori and for a positive integer $m$, there are well known formulas for the computation of the numbers $N(f^m)$, and for the Nielsen type periodic point numbers $NP_m(f)$ and $N\Phi_m(f)$ ([1], [2], [3]), the reader may be wondering what exactly is the point of this talk.

To answer this, we note first that we are wanting to compute these numbers for all positive integers $m$, and not just for some fixed $m$. Our aim, when the maps are periodic, is to reveal the fascinating patterns and interconnections that occur within these numbers, and to give shortcuts as to their determination. We show that in order to do this, we need only a relatively few easy computations of powers of the primes in the prime decomposition of $n$, the order of the given periodic map. We develop the theory of these patterns and shortcuts which were initially discovered by running simple computer programs. In this way the patterns can be discerned even when things get large enough or complicated enough, that we run out of computer memory.

The most easily discerned patterns occur in primitive periodic maps. A map $f$ is primitive if its linearization $F$, has only primitive $n$th roots of unity. For periodic maps that are not primitive, we use the eigenspace structure of the linearization $f$ to decompose it into a series of fibre preserving maps of trivial fibrations. We then use known fibre space techniques ([4]) to build up the three Nielsen theories of $f$ inductively by adding one primitive component at a time.

**References**


Obstruction theory for coincidences of multiple maps

Thaís Monis\textsuperscript{1} and Peter Wong\textsuperscript{2}

Abstract

Let \( f_1, ..., f_k : X \to N \) be maps from a complex \( X \) to a compact manifold \( N \), \( k \geq 2 \). In previous works \cite{Biasi2015, MonisSpiez2015}, a Lefschetz type theorem was established so that the non-vanishing of a Lefschetz type coincidence class \( L(f_1, ..., f_k) \) implies the existence of a coincidence \( x \in X \) such that \( f_1(x) = ... = f_k(x) \). In this paper, we investigate the converse of the Lefschetz coincidence theorem for multiple maps. In particular, we study the obstruction to deforming the maps \( f_1, ..., f_k \) to be coincidence free. We construct an example of two maps \( f_1, f_2 : M \to T \) from a sympletic 4-manifold \( M \) to the 2-torus \( T \) such that \( f_1 \) and \( f_2 \) cannot be homotopic to coincidence free maps but for any \( f : M \to T \), the maps \( f_1, f_2, f \) are deformable to be coincidence free.

References

\begin{enumerate}
\end{enumerate}

\textsuperscript{1}UNESP Rio Claro. \texttt{tfmonis@rc.unesp.br}

\textsuperscript{2}Bates College. \texttt{pwong@bates.edu}
Abstract

A single-valued self-map $f$ of a connected finite polyhedron $X$ can be lifted to an $n$-valued self-map $\phi_{p,f}$ of a covering space $p : \tilde{X} \to X$ of degree $n$. This construction produces $n$-valued maps of graphs, handlebodies, free $G$-spaces and nilmanifolds. A formula, that depends on the fundamental group endomorphism induced by $f$ and the monodromy action of the covering space, that relates the Nielsen numbers implies that $N(f) \leq N(\phi_{p,f}) \leq n \cdot N(f)$. For maps of graphs, a recent result of Jiang, Wang and Zhang is used to prove that if $f$ satisfies an appropriate generic remnant condition, then $N(\phi_{p,f}) = n \cdot N(f)$.

---

1University of California, Los Angeles. rfb@math.ucla.edu
(H, G)-Coincidence theorems and a Topological Tverberg type theorem for manifolds

Denise de Mattos¹ and and Edivaldo L. dos Santos² and Taciana O. Souza³

Abstract

Let $X$ be a paracompact space, let $G$ be a finite group acting freely on $X$ and let $H$ a cyclic subgroup of $G$ of prime order $p$. Let $f : X \to M$ be a continuous map where $M$ is a connected $m$-manifold (orientable if $p > 2$) and $f^*(V_k) = 0$, for $k \geq 1$, where $V_k$ are the Wu classes of $M$. Suppose that $\text{ind}_X \geq n > (|G| - r)m$, where $r = \frac{|G|}{p}$. In this work, we first estimate the cohomological dimension of the set $A(f, H, G)$ of $(H, G)$-coincidence points of $f$. Also, we give similar estimate in the case that $H$ is a $p$-torus subgroup of a particular group $G$ and as application, we prove a topological Tverberg type theorem for any natural number.

References


¹Departamento de Matematica, Instituto de Ciencias Matematicas e Computacao, Universidade de Sao Paulo. deniseml@icmc.usp.br
²Departamento de Matematica, Universidade Federal de Sao Carlos, Centro de Ciencias Exatas e Tecnologia. edivaldo@dm.ufscar.br
³Universidade Federal de Uberlandia-UFU. taciana@famat.ufu.br
REMARKS ON THE FIXED POINT PROPERTY FOR FLAG MANIFOLDS

Thaís F.M. Monis

Abstract

When $K$ is $\mathbb{R}$, $\mathbb{C}$ or $\mathbb{H}$, let $U_K(n)$ denote the group of $n \times n$ orthogonal, unitary, or symplectic matrices, respectively. The projective space $K P^n$ is obtained as the quotient

$$\frac{U_K(n+1)}{U_K(1) \times U_K(n)}$$

and it is well-known that for $K = \mathbb{R}$ or $\mathbb{C}$, $K P^n$ has the fixed point property if and only if $n$ is even. A generalization of the projective spaces are the flag manifolds

$$K M(n_1, \ldots, n_k) = \frac{U_K(n)}{U_K(n_1) \times \cdots \times U_K(n_k)},$$

with $n = n_1 + \cdots + n_k$.

In [2], the authors have proved that if $K M(n_1, \ldots, n_k)$ has the fixed point property then $n_1, \ldots, n_k$ are distinct integers and, if $K = \mathbb{R}$ or $\mathbb{C}$, at most one is odd. The conjecture is that the converse is also true. Such conjecture also appears in [1] and in [3] for the particular class of the grassmanian manifolds.

In this talk, we intend to approach the relationship between the fixed-point property of a flag manifold and a Borsuk-Ulam property.

References


1Univ. Estadual Paulista. tfmonis@rc.unesp.br
Abstract

For a given group $G$ and its automorphism $\varphi$ one defines an equivalence relation by the following rule: two elements $x, y \in G$ are called twisted $\varphi$-conjugated if there exists an element $z \in G$, such that $x = zy\varphi(z^{-1})$. This relation divides a group onto $\varphi$-conjugacy classes. The number of this classes is called Reidemeister number of the automorphism $\varphi$ and is denoted by $R(\varphi)$.

Twisted conjugacy classes in groups are useful in Nielsen-Reidemeister fixed point theory. Let $f : X \to X$ be a homeomorphism of a compact topological space $X$ on itself, and $\varphi$ be the automorphism of $\pi_1(X)$ induced by the homeomorphism $f$. Then the number $R(\varphi)$ is closely connected with the Nielsen number $N(f)$ and Reidemeister number $R(f)$, which are the main objects of study in Nielsen-Reidemeister fixed point theory.

In the talk we are going to discuss some properties of groups, involving twisted conjugacy classes: the $R_\infty$ and the $S_\infty$-properties, dependence of structure of a group from the number of twisted conjugacy classes, connections between properties of the twisted conjugacy class of the unit element and properties of a group.

A big part of the results which we are going to talk about are obtained jointly with A. Felshtyn \cite{1}.

References

\cite{1} A. Felshtyn, T. Nasybullov, The $R_\infty$ and $S_\infty$ properties for linear algebraic groups, to be published in J. Group theory (2016).

\footnote{University of Bologna. ntr@math.nsc.ru, timur.nasybullov@mail.ru}
COINCIDENCES AND SECONDARY NIELSEN NUMBERS

Ulrich Koschorke

Abstract

Let $f_1; f_2 : X^m \to Y^n$ be maps between smooth connected manifolds of the indicated dimensions $m$ and $n$. Can $f_1; f_2$ be deformed by homotopies until they are coincidence free (i.e. $f_1(x) \neq f_2(x)$ for all $x \in X$)? The main tool for addressing such a problem is traditionally the (primary) Nielsen number $N(f_1; f_2)$. E.g. when $m < 2n - 2$ the question above has a positive answer precisely if $N(f_1; f_2) = 0$. However, when $m = 2n - 2$ this can be dramatically wrong, e.g. in the fixed point case when $m = n = 2$. Also, in a very special setting the Kervaire invariant appears as a (full) additional obstruction.

In this paper we start exploring a fairly general new approach. This leads to secondary Nielsen numbers $SecN(f_1; f_2)$ which allow us to answer our question e.g. when $m = 2n - 2$; $n \neq 2$ is even and $Y$ is simply connected.
The Borsuk–Ulam problem for homotopy class of functions. An approach using braid groups
Vinicius Casteluber Laass¹

Abstract

The well-known Borsuk-Ulam theorem states that given any continuous map \( f : S^2 \to \mathbb{R}^2 \), there is a point \( x \in S^2 \) such that \( f(-x) = f(x) \). One possible generalization of this theorem is to consider other spaces and involutions. More precisely, if \( \tau : M \to M \) is a free involution, we say that a triple \((M, \tau; N)\) has the Borsuk-Ulam property if for every continuous function \( f : M \to N \), there is a point \( x \in M \) such that \( f(\tau(x)) = f(x) \). So if \((M, \tau; N)\) does not have the Borsuk-Ulam property, there exists a continuous map \( f : M \to N \) such that \( f(\tau(x)) \neq f(x) \) for every \( x \in M \). A natural question that arises is the following: up to homotopy, how many functions are there with this property? In this exposition, I will show a relation between this problem and a diagram involving braid groups, in the case that \( M \) and \( N \) are compact surfaces without boundary, and I will give some examples of functions from Torus to Klein bottle.

References


¹Universidade Federal da Bahia. vinicius.laass@hotmail.com
Topological complexity of spaces with the presence of actions of finite groups

Wacław Marzantowicz

Abstract

In order to measure complexity of the process of motion planning of a mechanical system (a robot), M. Farber [Fa1], [Fa2], [Fa3] introduced the notion of topological complexity, $TC(X)$, of a topological space $X$. This is defined as the minimal number of domains of continuity whose union covers $X \times X$, where “domain of continuity” is taken to mean an open subset $U \subseteq X \times X$ such that a motion planning algorithm exists over $U$ (i.e., there exists a continuous function $s: U \rightarrow PX$ such that $s(x, y)(0) = x$ and $s(x, y)(1) = y$ for any pair $(x, y) \in U$). Topologically, this corresponds to the Švarc category of the path fibration $PX \rightarrow X \times X$. Due to its applications in topological robotics — its knowledge is of practical use when designing optimal motion planners — and close relation to Lusternik–Schnirelmann category, topological complexity has attracted plenty of attention in recent years.

Mechanical systems often come equipped with symmetries visible in their configuration spaces, thus it not surprising that there have been attempts at weaving symmetries into the definition of topological complexity. We would like to present properties of these “symmetric” invariants. In fact, there are at least four different approaches, depending on how one decides to interpret the additional structure. One can rigidify planners ([Co-Gr]), locally simplify them at the cost of increasing the global amount of domains of continuity ([Lu-Mar]), simplify motion planners globally ([Bl-Kal2]), or “stabilize” them by twisting via $X \times_G EG$ ([Dr1]). There are already several computations of these invariants for various $G$-spaces ([Bl-Lü-Zi], [Bl-Kal1], [Bl-Kal2], [Dr1], [Fr-Pa], [Go-Gra-To-Xi]), as well as relating them to classical invariants of transformation group theory ([Bl-Kal1]) we would like present in brief.

References


1UAM Poznań, Poland
[Co-Ed-Ha-Na] Cole-McLaughlin, K.; Edelsbrunner, H.; Harer, J.; Natara- 
jan, V.; Pascucci, V. Loops in Reeb graphs of 2-manifolds, Discrete Com- 

[Cor] O. Cornea et. al., Lusternik-Schnirelmann Category, AMS, Math. 

[Co-Gr] H. Colman, M. Grant, Equivariant topological complexity, Alg. and 


[Dr1] A. Dranishnikov, On topological complexity of twisted products, Topo- 

[Dr1] A. Dranishnikov, The topological complexity and the homotopy cofiber 
of the diagonal map for non-orientable surfaces, arXiv:1506.06291v6

[Dr-Ka-Ru] A. Dranishnikov, M. G. Katz, and Y. B. Rudyak, Small values 

[Fa1] M. Farber, Topological complexity of motion planning, Discrete Com- 

[Fa2] M. Farber, Topology of robot motion planning, Morse theoretic meth- 
ods in nonlinear analysis and in symplectic topology, ed. O. Cornea, 
drecht, 2006.

[Fa3] M. Farber, Invitation to topological robotics, Zurich Lectures in Ad- 
vanced Mathematics. (EMS), Zürich, 2008.

[Fa-Gra] M. Farber, M. Grant, Symmetric motion planning, Topology and 

[Fa-Ta-Yu] M. Farber, S. Tabachnikov, S. Yuzvinsky, Topological robotics: 
motion planning in projective spaces, Int. Math. Res. Not. 2003, no. 34, 
1853–1870.

[Fr-Pa] A. Franc, P. Pavešić, Lower bounds for topological complexity, Topo- 

Schnirelmann category of 3-manifolds, Topology 31 (1992), no. 4, 
791–800.

[Go-Gra-To-Xi] J. González, M. Grant, E. Torres-Giese, M. Xicoténcatl, 
Topological complexity of motion planning in projective product spaces, 

of invariant functions on closed orientable surfaces, Bol. Soc. Mat. Mex. 

Z. 146 (1976), 143-148.

of the Reeb graph as a sub-complex of manifold, Topol. Methods Nonlinear 

[Lu-Mar] W. Lubawski, W. Marzantowicz, Invariant topological complexity, 

[Mar] W. Marzantowicz, A G-Lusternik-Schnirelman category of space with
Periodic points on Torus fiber bundles over the circle

Weslem Liberato Silva\textsuperscript{1} and Rafael Moreira de Souza\textsuperscript{2}

Abstract

Let $M$ be a fiber bundle with base $S^1$ and fiber torus and $f : M \to M$ a fiber-preserving map over $S^1$. In this work we investigate when there exists a map $g$ fiberwise homotopic to $f$ such that $g^n : M \to M$ is a fixed point free map, where $g^n$ denotes the composed of the $g$ by itself $n$ times. To ensures the existence of a such map $g$ we will present necessary and sufficient conditions in terms of induced homomorphism $f^n_\#$.

References


\textsuperscript{1}Universidade Estadual de Santa Cruz wlsilva@uesc.br
\textsuperscript{2}Universidade Estadual de Mato Grosso do Sul moreira@uems.br
COMMON VALUE CLASSES

Xuezhi Zhao

Abstract

The famous Whitney embedding theorem said that any $n$-manifold can be embedded smoothly into $R^{2n+1}$. His more sensitive result states that any $n$-manifold can be embedded (may not smoothly) into $R^{2n}$ [1]. The key definition of self-intersection points was introduced there. We generalized Nielsen fixed point theory into the case of intersection points. A new invariant is defined to estimate the number of intersection points of any two maps $f, g : X \to Y$, based on the definition of fixed point classes. Since we consider the point in domain $X$ other than target $Y$, we prefer to say common value pairs. The case of self-intersection is also addressed.

References


\textsuperscript{1}Department of mathematics, Capital Normal University
zhaoxve@mail.cnu.edu.cn
Posters

Remarks about the Borsuk-Ulam theorem for some 4-closed manifolds ..................30
Anderson Paião dos Santos

Geometric procedures to work with Nielsen Borsuk–Ulam classes .......................31
Fabiana Santos Cotrim and Daniel Vendrúscolo

On the Nielsen coincidence number for selfmaps on Sol torus bundles ..................32
Karen Regina Panzarín

Grids and branched coverings of the sphere ..................................................33
Natalia A. Viana Bedoya

Remarks about the Borsuk-Ulam theorem for some 4-closed manifolds

Anderson Paião dos Santos

Abstract

Let $(X, \tau)$ be a space $X$ with a free involution $\tau$ on $X$. We say that the Borsuk-Ulam theorem holds for the triple $(X, \tau; \mathbb{R}^n)$ if for any continuous map $f: X \to \mathbb{R}^n$, there exists a point $x \in X$ such that $f(x) = f(\tau(x))$.

Now, let $M$ be homotopy equivalent to the total space of a $F$-bundle over $B$, where the base $B$ and the fiber $F$ are closed surfaces, and consider a non-trivial class $[\varphi] \in H^1(M; \mathbb{Z}_2)$. In this work we present some results about the Borsuk-Ulam theorem for triples $(M_\varphi, \tau_\varphi; \mathbb{R}^n)$, where $M_\varphi$ is the double covering of $M$ and $\tau_\varphi$ is the free involution on $M_\varphi$ such that $M_\varphi/\tau_\varphi = M$ in the case which the fiber $F = S^2$ or $\mathbb{R}P^2$ and the base $B$ is $K(\pi, 1)$.

References


---

1Universidade Tecnológica Federal do Paraná. anderpaiao@gmail.com
GEOMETRIC PROCEDURES TO WORK WITH NIELSEN BORSUK–ULAM CLASSES

Fabiana Santos Cotrim$^1$ and Daniel Vendrúscolo$^2$

Abstract

In a recent work it was defined the Borsuk-Ulam Nielsen classes for maps from an orientable manifold with a free involution to a manifold of the same dimension, more specifically:

**Definition** [1, 4.1]: Let $(X, \tau; Y)$ be a triple where $X, Y$ are finite $n$-dimensional complexes, $\tau$ is free simplicial involution on $X$ for any map $f : X \to Y$ with $\text{Coin}(f, f \circ \tau) = \{x_1, \tau(x_1), \ldots, x_m, \tau(x_m)\}$ we define the Borsuk-Ulam coincidence set for the map $f$, as the set of pairs:

$$\text{BUCoin}(f; \tau) = \{(x_1, \tau(x_1)); \ldots; (x_m, \tau(x_m))\}$$

and we say that two pairs $(x_i, \tau(x_i)), (x_j, \tau(x_j))$ are in the same BU-coincidence class if there exist a path $\gamma$ from a point in $\{x_i, \tau(x_i)\}$ to a point in $\{x_j, \tau(x_j)\}$ such that $f \circ \gamma$ is homotopic to $f \circ \tau \circ \gamma$ with fixed endpoints.

In the same work it was defined the signature of such a class with is a sequence of integers (the local index of the coincidences in the class) and using three procedures in the signature: split, join and blend it was presented conditions for such a class to be essential.

In this poster we will present the geometric versions of such procedures. Such procedures can be used to define a Nielsen-Borsuk-Ulam number and to prove the realization of such number and some conditions.

References


---

$^1$CCN-UFSCar. fabiana_cotrim@yahoo.com.br

$^2$DM-UFSCar. daniel@dm.ufscar.br
ON THE NIELSEN COINCIDENCE NUMBER FOR SELFMAPS ON SOL TORUS BUNDLES

Karen Regina Panzarin

Abstract

We compute the Nielsen coincidence number $N(f, g)$ of selfmaps $f, g : M_A \to M_A$ of an orientable torus bundle over $S^1$, $M_A$ with an Anosov matrix $A$ in $SL(2, \mathbb{Z})$. We begin with self homeomorphisms which yield a result that coincides with the one obtained in fixed point theory where we obtain either $N(f, g) = 0$ or $N(f, g) = 4$, and extend to general selfmaps $f, g : M_A \to M_A$. The results that we get depend on the degrees of the induced maps $\bar{f}, \bar{g} : S^1 \to S^1$.

References

In this work we study \( m \times n \) grids of nodes and edges realizing branched coverings of the sphere. We will introduce this relation and their properties.

References


\(^1\)Departamento de Matemática, Universidade Federal de São Carlos–SP, Brazil. naviana@gmail.com