Humans-with-Media: Twenty-Five Years of a Theoretical Construct in Mathematics Education

Marcelo C. Borba, Daise Lago P. Souto, José Fernandes T. Cunha, and Nilton S. Domingues

Contents

Abstract

In this chapter, we discuss the genesis, the epistemological vision, and how the humans-with-media construct has been developed in dialog with activity theory, social semiotics, and other theories. We show how the construct has been transformed by different studies and its relation to different media. We also present research that relied on this theoretical construct to understand how digital technologies cause modifications in the production of mathematical knowledge. We address theorizations and situate the notion of the agency of media within the humans-with-media construct. We discuss how the power of the virus transformed classrooms – and society in general – during the pandemic period, as well as the challenges of doing education during and after the COVID-19

M. C. Borba (✉) · N. S. Domingues
State University of São Paulo (UNESP), Rio Claro, Brazil
e-mail: marcelo.c.borba@unesp.br

D. L. P. Souto · J. F. T. Cunha
State University of Mato Grosso (UNEMAT), Barra do Bugres, Brazil

B. Pepin et al. (eds.), *Handbook of Digital Resources in Mathematics Education*,
Springer International Handbooks of Education,
https://doi.org/10.1007/978-3-030-95060-6_7-1

pandemic. It is emphasized how this construct may be helpful in the struggle for social justice within and beyond mathematics education.

Keywords

Humans-with-media · Agency of media · Digital technologies · Activity theory · Social inequality

Introduction

It has been more than 20 years since the theoretical construct of humans-with-media was developed. Since the mid-1990s, this notion has emphasized, epistemologically, how media and humans are active in the production of knowledge. Ontologically, this notion emphasizes how available technology can constitute what it means to be human. Borba and Villarreal (2005) conducted a first synthesis of the construct. Since then, it has been used in countries such as Portugal to deal with the collective way a student-with-media engages in problem-solving activities (Jacinto and Carreira 2017). In Colombia, Santa Ramirez (2016) extended the notion of media to paper folding, arguing that collectives of human-with-paper folding generate mathematics in different ways. These kinds of arguments theoretically and empirically articulate ideas related to how mathematics is imbued with media. Thus, mathematics is seen as a product that is historically and culturally marked by different media. Orality, writing and computer science, and the technologies of intelligence (Lévy 1993) are coproducers of the knowledge generated by humanity. Artifacts and media are the result of knowledge and dialectically the source of new knowledge.

The notion of humans-with-media has its origins in the philosophy of technique (Lévy 1993), activity theory (Engeström 1987), and phenomenology (Schütz 1970). From the philosophy of technique, it brings the idea of the thinking collective, from activity theory the idea of reorganizing thinking from the interaction of humans with digital technologies, and from phenomenology the idea that the world and humans are part of a single whole. Two transformations have taken place in recent years. First, a revisiting of activity theory, this time of the third generation, has led us to absorb the triangles of the Helsinki school, which allows us to talk more specifically about learning. However, it also allows us to see the humans-with-media construct as an activity system, originally proposed by Engeström (1987, 1999). Second, we outline the idea of a possible fourth generation of activity theory – or a ramification of the third – as we break the rigidity of the triangles and see artifacts as subjects or community, for example, Souto and Borba (2016, 2018).

Activity theory provided theoretical support for research to identify motives and understand the impact of transforming the activity systems of students, teachers, and technologies, in short, on aspects of the learning process. However, the analysis of videos as ready-made digital artifacts encountered limitations with this theory. To overcome this situation, the notions, ideas, and perspectives of social semiotics

(Halliday 1993; O'Halloran 2011, 2015) proved adequate. Semiotic resources and their different combinations (multimodality) consider the production of meanings to be fundamentally related to the social aspects that shape individuals and societies. These concepts present in social semiotics align with the construct of humans-with-media.

Additionally, inherent in the humans-with-media construct is the notion that media have agency. More recently, with the pandemic, Borba (2021) proposed and discussed how such agency should be extended to things such as SARS-CoV-2 and houses by bringing homes into the collective of humans-with-media. Borba made clear the political role of the notion of humans-with-media, for example, in the abysmal difference between homes that may or may not allow for equal opportunity in mathematics education.

Throughout this chapter, we systematically expand discussions about the humans-with-media theoretical construct, starting with its genesis, followed by its theoretical basis, applications in mathematics education research, and transformations based on interrelationships with activity theory and social semiotics. We show how the construct has been transformed by different studies and its relation with different media and researchers. With this, we seek to highlight the relevance of this construct to the understanding of the transformations of learning with digital technologies.

Media should be understood in a broad sense. Knowledge is produced by humans and nonhumans. Media may be a library, as suggested by Lévy (1993), or homes, as evoked by Borba (2021). In this sense, computers, videos, paper-and-pencil, regular classrooms, homes, and libraries are examples of media. In particular cases, researchers who work within this perspective have referred to collectives of humans-with-software-Internet and humans-with-videos, in order to highlight the agency of nonhuman actors. In addition to these expressions, where the word media is replaced by examples of digital technologies, we point out that throughout this chapter, expressions such as humans-with-homes, humans-with-things, and other variations, also refer to collectives of humans-with-media. We use these variations to emphasize that things have agency, as discussed in this chapter.

Finally, we will present the development of humans-with-media during the pandemic. The construct was fundamental in handling the situation of mathematics education during the pandemic, and dialectically, the concept made evident new issues that can be translated as collectives of humans-with-things and humans-with-homes, showing the fundamental role of both the virus and homes for mathematics education.

The Humans-with-Media Construct

The notion of humans-with-media has an underlying epistemological vision that has been developed over decades. Cultural, social, and emotional issues are some elements that permeate it and contribute to the understanding of how technologies participate and influence the processes of knowledge production. Regarding the latter, Borba et al. (2022) emphasized that there are distinct types of visions.

> [. . .] those more skeptical that reject the presence of any digital technology for considering that they bring damage to the reasoning and, therefore, make us 'shallow' people. [. . .] [others] who admit the presence of these technologies, but see them in a conservative way, as mere tools capable of complementing or even replacing human work. [. . .] Finally, there are the more contemporary ones, which, unlike the others, conceive technologies as extensions of the human body, of our senses, or as something dynamic and fluid that is produced based on a collective thought that aims primarily at obtaining experience and effect. (pp. 65–66)

It is in this latter, more contemporary view that the humans-with-media construct is situated. It assumes that human and nonhuman actors (such as technologies) constitute each other. Both are agents that are intertwined in such a way that they form a collective that "thinks together."

In this section, we will introduce the roots of the construct, and then we will show the generative nature of this notion. We will then show the fundamental change that is a reunion of this construct with activity theory, which was also part of its origins. If the notion of reorganization, coming from activity theory, was key to the construct, then the humans-with-media notion seems to be relevant to breaking the rigidity of the triangles that structure such theory.

Digital Technology Phases and the Humans-with-Media Construct

The humans-with-media construct considers digital technologies to be coparticipants in the production of mathematical knowledge. The development of this construct appears associated with changes in digital technology itself. Thus, we believe it is important to present a synthesis of how the use of these technologies in mathematics education has been characterized throughout different phases. Borba (2012) and Borba et al. (2014, 2016, 2022) proposed such phases.

> The phases have been appropriated by part of the mathematics education community to talk about different aspects of the use of digital technologies in the "new mathematics classroom" or even to think about how digital technologies transform or are constitutive of mathematics education. (Borba et al. 2022, p. 19)

The purpose of dividing this technology use into phases was to classify the way that practices were transformed as technologies entered the classroom and other spaces of mathematics education. Borba et al. (2022) made it clear that the phases, while having distinct elements, are also interconnected and cannot be seen as disjoint sets. Borba et al. (2014) presented the aspects and elements that characterized the first four phases (see Table 1).

The first phase began when the Logo software started to be used in mathematics education. Its functionalities, such as the turtle's steps and spins that were made possible by programming commands, made it possible to build geometric objects.

Table 1 Four phases of digital technologies in mathematics education

Phase	Technologies	Nature or technological basis of activities	Theoretical perspectives or notions	Terminology
First phase (1985)	Computers; simple and scientific calculators	LOGO programming	Constructionism; microworld	Computer technologies (IT)
Second phase (early 1990s)	Computers (popularization); graphing calculators	Dynamic geometry (Cabri Géomètre; Geometricks); multiple function representations (Winplot, Fun, Mathematica); CAS (Maple); games	Experimentation, visualization and demonstration; risk zone; connectivity; constructionist learning cycle; humans-with-media	IT; educational software; educational technology
Third phase (1999)	Computers, laptops, and Internet	Teleduc; e-mail; chat; forum; Google	Online distance education; online interaction and collaboration; learning communities	Information and communication technologies (ICT)
Fourth phase (2004)	Computers; laptops; tablets; cell phones; fast Internet	GeoGebra; virtual learning objects; Applets; videos; YouTube; WolframAlpha; Wikipedia; Facebook; ICZ; Second Life; Moodle	Multimodality; telepresence; interactivity; Internet in the classroom; online video production and sharing; digital mathematical performance	Digital technologies (TD); mobile or portable technologies

Source: Borba et al. (2014, p. 39). Reproduced with permission from the author

Beyond the classroom, the initiatives to introduce computer labs in schools were also characteristic of this phase (Borba et al. 2014). The second phase was marked by the use of software to represent multiple geometric functions and dynamic geometry. The graphical interfaces of these software allowed us to dynamize geometric constructions by "dragging" with the mouse. According to Borba et al. (2014), the mathematical experiments performed with this software enabled "new ways to think with technologies" (p. 27).

Notably, the transition from one phase to another does not imply that digital technologies used in one phase are no longer used in the next. In this regard, Borba et al. (2016) argued that "we were still trying to understand the role of computer labs in schools, a novelty of the first and second phases (depending on the country), when the Internet appeared" (p. 590).

The presence of the Internet characterized the third phase of digital technologies, which brought about qualitative changes in the ways of communicating and relating to each other. The Internet enabled the development of online courses in which teachers and students could interact remotely. At that stage, online collaborative learning became an emerging research topic (Borba et al. 2016, 2022).

Substantial changes in the speed of the Internet have made the online environment more dynamic. Thus, the available content began to be produced not only by developers but also by users of this technology. This new generation of the Internet – which has become known by various denominations like Web 2.0 or fast Internet – is the symbol of the fourth phase of digital technologies (Borba et al. 2014, 2016). This phase "brings us greater opportunities for collaborative learning and personalization of the Internet through personal devices–and opens up the possibility that everyone with Internet access can express themselves digitally through different forms of multimodal discourse" (Borba et al. 2016, p. 590).

Although fast Internet is a key term in the fourth phase, Borba et al. (2014) highlighted several aspects that are also present, such as the use of smartphones, and the production of digital videos, the ease of storing them, the ability to share them on platforms such as YouTube, social networks, and virtual learning environments.

Unlike the other phases, which can be represented by technological artifacts, the fifth phase is marked by the transformations brought about by the impact of SARS-CoV-2. According to Borba et al. (2022), this virus "has influenced the presence of digital technologies in Mathematics Education to an intensity that no human-designed (or humans-with-technologies) program has achieved" (p. 26).

The Roots and Epistemological View of the Humans-with-Media Construct

We have already discussed how the humans-with-media construct has roots in activity theory (Tikhomirov 1981). The focus is how it is transformed by the third generation of activity theory and at the same time transforms activity theory. If knowledge is produced by collectives of humans and nonhumans, we argue that nonhumans also have agency. In this sense, we propose that we may have a fourth generation of activity theory in which the triangles (Fig. 1a) that are static on paper in the third generation may become dynamic (Fig. 1b).

This theoretical construct conceives the process of knowledge production as something collective and complex that goes beyond biological beings and includes technologies as mobilizing and influencing agents. It also considers cultural, social, and economic issues. Several examples in our research show how different technology may have agency to make learning possible, or in a more general way, how, for example, mobile phones constitute what humans are in this century. Humans develop technology, and in its turn, technology developed in different historical moments shaped how humans are constituted. The collective epistemological actor as defended by Borba (1993, 1999), which includes humans and nonhumans, is fluid, dynamic, and results from the anthropomorphization of technologies (or media). Theorists such as Schütz (1970), Lévy (1993), Lave (1988), Wagner (1979), and Latour (2001, 2020) are inspirations behind this nonseparation of humans and nonhumans.

(A) (B)

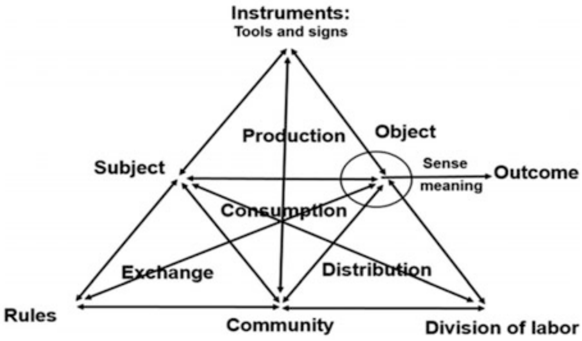

Fig. 1 Representation of an activity system (Source: Adapted from Borba (2021, p. 393). Reproduced with permission from the author)

In Borba (1993), there are indications of what would become the notion of humans-with-media. Entering his doctoral program at Cornell University, which was filled with computers everywhere, the author transitions from the mathematical world of pencil and paper to the mathematics of software that focuses on function (Function Probe). The ways by which mathematics was transformed by the use of computers is reinforced by his research when he analyzes in detail the knowledge produced by high school adolescents. The first author of this chapter then dialogues with his previous work where mathematics was modeled by the orality of illiterate adults and semi-illiterate children in a favela (Borba 1987, 1988, 1993).

It seems that contact with the author Lévy (1993) was opportune, since he described the technologies of intelligence as orality, writing, and informatics. This author, among others, presented ideas of a cognitive ecology and a thinking collective that underpin the idea that knowledge is produced in a collective way, which is supported by several epistemological views. In our case, however, it is embodied in a different way: it is collective and performed by humans and nonhumans.

The hyphens that make up the expression humans-with-media come from phenomenology, in particular from Schütz's Phenomenology, as summarized by Wagner (1979). From this perspective, humans are seen as beings-in-the-world-with-the-others. In balancing realism and idealism, phenomenology proposes that neither the human being nor the world comes first. They do not exist without each other. The "Other" can be a human or not. We are with things, with humans, and with other living beings. We-are-with-media! This is the origin of different terminologies like human-being-with-technology, human-technologies, and the one that became consolidated: humans-with-media.

The notions of intershaping relationships – humans mold (transform) the technology developed, and the technology used molds (transforms) humans – and of reorganization of thought crown this idea. The former is loosely inspired by the work of Lave (1988), who proposed that the supermarket interacts with mathematics that the shopper knows. The consumer in the supermarket does not use mathematics in a "pure" way, as they are also structured by the practice of buying: cognition in practice is the name of her book. Tikhomirov (1981) presented the notion that thought is reorganized by informatics media. With the use of computers, a "man-computer" system is formed, in which immediate feedback causes qualitative transformations in thought and the receipt of information from one by the other results in actions.

> The computer changes the structure of human intellectual activity. Memory, the storage of information, and its search (or reproduction) are reorganized. Communication is changed, since human communication with the computer, especially in the period when languages that are similar to natural language are being created, is a new form of communication. Human relations are mediated through the use of computers. (Tikhomirov 1981, p. 277)

According to Tikhomirov (1981), the way in which the process of thought reorganization occurs depends on the computer used, the fluency of the human being in relation to the potentialities of this technology, and the working conditions. The notions of an intershaping relationship (Borba and Villarreal 2005**)** and reorganization of thinking suggest that media have agency; however, in the literature, agency is emphasized as something inherent solely to human beings (e.g., Bandura 1989). Authors such as Kaptelinin and Nardi (2006) acknowledge the possibility of nonhuman actors, including nonliving beings, having agency. In line with these ideas, Latour (2020) suggested that a virus has agency, something that will be addressed later in this chapter. Currently, the reinterpretation of this notion of agency led Borba et al. (2022) to characterize the fifth phase of digital technologies, reaffirming that "technologies have the power to act and modify our culture" (p. 68). They are cultural products of human beings that dialectically shape themselves. Thus, it is possible to see how the humans-with-media construct emerges and has been enhanced with these theoretical foundations and in different phases of digital technologies.

Theoretical Contributions and Dialogues

The epistemological roots and vision that underpin the humans-with-media construct, presented earlier, suggest that "technology not only mediates from the outside what and how humans know, but they are part of the collective that knows" (Borba et al. 2022, p. 69). Over time, the construct has been a source of inspiration for other authors who have dialectically contributed to its development.

Borba (1999) and Borba and Villarreal (2005), during the transition from the third to the fourth phase of digital technologies, already stated that these technologies are protagonists in the process of knowledge production. The theorizations of this construct and dialogue with activity theory that are systematized in research – such as Souto (2013), Souto and Araújo (2013), and Souto and Borba (2016,

2018) during the fourth phase of digital technologies – perform the theoretical exercise of analyzing how this construct has been developing over time in a process analogous to that of activity theory. In these studies, there are reinterpretations of the scope of mathematics education in terms of the principle of expansive transformations and indications of how the intershaping relationship can contribute to the identification of the object of activity. To this end, Souto (2013) suggested searching for answers to the following questions: What do you want to study or approach? What is effectively being studied or addressed? How did the subjects mobilize themselves in search of overcoming and/or identifying innovative, alternative paths to solve a given situation? Why did they mobilize in such a way? Like this, these authors present a way to analyze systems proposed by Engeström (1987), illustrated in Fig. 1a, based on the foundations of the humans-with-media construct.

Originally, in the system proposed by Engeström (1987), triangles divide humans, things, and media into distinct vertices. This part of activity theory, tracing back several generations, contradicts ideas that are imbued in the humans-with-media construct. This contradiction can be rectified by "dynamic triangles": triangles that are not compatible with media writing and not supported by pencil and paper. Digital technologies – supported by computers – with their typical plasticity may be the way to display how this contradiction has been resolved. Thus, in a QrCode (Fig. 1b), one can see the media and the Internet occupying various vertices of the triangle. The interaction is qualitatively different. In this sense, if we think of generations in terms of models of triangles (Engeström 1999; Engeström 2001; Engeström and Sannino 2009), this dynamic view would lead to a fourth generation or a branch in the third generation, as discussed by Souto and Borba (2016, 2018).

In the activity system shown in Fig. 1a, the media move around and play more than one role. To illustrate how the Internet and media jump from one corner to another in this triangular representation, Borba (2021) proposed an animation in GIF format, which can be accessed through the QrCode (Fig. 1b) or with the link: https://igce.rc.unesp.br/#!/pesquisa/gpimem—pesq-em-informatica-outras-midias-e-educacao-matematica/animacoes/triangulo-sannino–engestrom/.

This new phase of the "dialogue" between activity theory and the human-with-media construct has resulted in important developments towards understanding learning. With that, Galleguillos (2016), Cunha (2018), Domingues (2020), and Borba (2021), near the end of the fourth phase of digital technologies, intensified the relationship between humans-with-media and different aspects of activity theory. They focused on understanding how learning can occur in collectives with a given type of digital technology, and they supported their research through issues related to the discursive manifestations of internal contradictions and the formation of networks of activity systems. For example, Galleguillos (2016) identified the ways by which the dialectical relationship between humans and technologies provokes tensions, dilemmas, conflicts, dead ends, and double bindings. In the research by Galleguillos (2016), these conflicts culminated in internal contradictions during mathematical modeling problem-solving, which reaffirms media protagonism in teacher-with-media collectives that may or may not lead to learning.

Activity systems (Fig. 1) do not exist in isolation, because each of the participants (human and nonhuman) is, simultaneously or not, part of more than one activity system, either in their respective institutions, in the classroom, in study or research groups, or among others. All these systems have interconnected elements (Engeström 2001), which form networks of activity systems (Fig. 2). Media protagonism is also observed when these networks are created, as investigated by Cunha (2018) in a continuing education course developed in a hybrid model. In this course, digital technologies such as the Internet and virtual learning environment were used (third phase), as well as cartoon-style digital videos (fourth phase).

By verifying that the systems were interrelated, Cunha (2018) proposed a representation for a network of activity systems (Fig. 2), which was inspired by the format of mesh networks. Mesh networks are a type of network of digital devices in which nodes are interconnected to each other through various routes on different media, either wired or wireless (Zhao et al. 2006). Therefore, the purpose is to show that analogous movement or any kind of action in one activity system can interfere with other systems connected to that network.

As shown in Fig. 2, the network is made up of several activity systems. However, Cunha (2018) analyzed only three of them, namely: (i) the encapsulated training system, which refers to the predominant model of continuing education prior to developments in the research; (ii) the idealized blended-learning system, representing the proposed course; and (iii) the activity system of a group of teachers. The latter was created as the teachers dealt with the tasks proposed in the course. According to Cunha (2018), the systems, when sharing their object, were influenced by each other. These interrelationships, which resulted in mutual transformations, suggest the occurrence of an intershaping relationship process, as is emphasized by the humans-with-media construct.

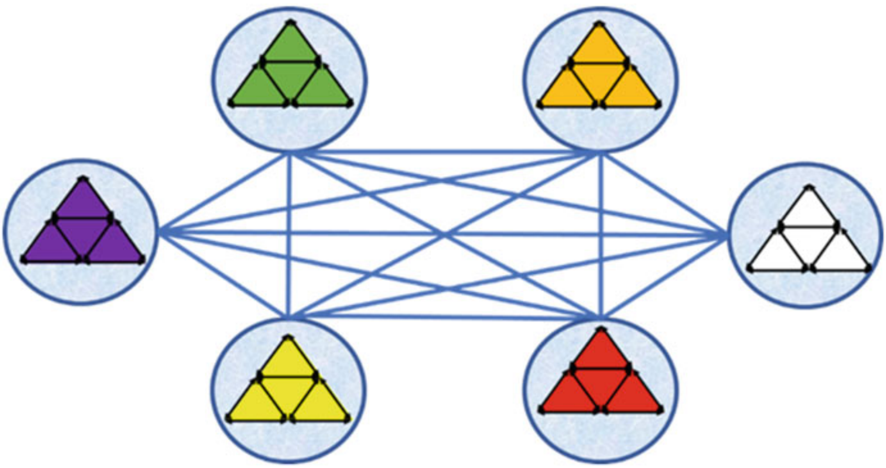

Fig. 2 Representation of a network of activity systems (Source: Cunha (2018, p. 73). Reproduced with permission from the author)

The research by Cunha (2018) also pointed out that the digital technologies used during the proposed course caused internal contradictions in the analyzed activity systems. These contradictions resulted in modifications in the objective, rules, community, and organization of work in the activity system created by the group of teachers. By suggesting that digital technologies can influence movements in activity systems, the results of this research are in line with the ideas of authors such as Souto and Borba (2016, 2018) and Borba (2021), who argue that media are not just artifacts; they have power to act and operate as a subject in activity systems.

In harmony with these ideas, Domingues (2020) also conducted an analysis of interconnected system networks, focusing on internal contradictions. In this investigation, the humans-with-media construct itself is analyzed as an activity system. This system (Fig. 3a), devised by Souto (2013), includes a new element: the study proposal.

The data from Domingues (2020) were produced in the context of the first Festival of Digital Videos and Mathematics Education. This festival is a large-scale educational event that impacts teachers, students, and school communities at all educational levels in Brazil. The event has taken place annually since 2017. According to Borba et al. (2022), the festival:

> Is not only a research locus, but a virtual space that brings the classroom closer to society as a whole. [. . .] Students, with the guidance of their teachers, were invited to produce videos that could even result in part of the material to be studied, becoming, then, co-authors of the school curriculum. (p. 52)

Thus, the festival becomes a stage for students to critically express reflections about real problems through videos, such as the environmental tragedy that occurred in the city of Brumadinho (MG), Brazil. This tragedy is pictured in the video "Sea of mud: Modelling and Mathematics Education," an awarded video in the third edition

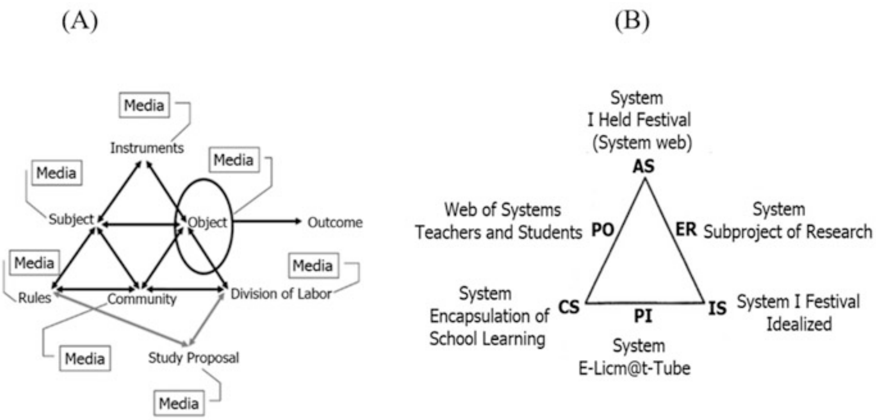

Fig. 3 Humans-with-media construct as an activity system (Source: Domingues (2020, p. 63, 92). Reproduced with permission from the author)

of the festival (Available at: https://youtu.be/YpCteGqjxd0). The festival integrates teaching, research, and outreach activities.

Discussions like the one depicted in this video are fostered by the critical mathematics education trend (Frankenstein 1987; Powell and Frankenstein 1997; Skovsmose and Borba 2004). This trend focuses on problems of social inequalities and cultural, political, economic, environmental, and other asymmetries in mathematics education and challenges more "conservative" views of mathematics as something antiquated (Borba et al. 2022). We observe this trend in the video discussed earlier, when elementary school students denounce how the tragedy caused by the collapse of a mining company's barriers has terribly impacted the environment and the entire local community.

If once more we consider the research by Domingues (2020), we find that he relied theoretically on the concepts of reciprocal shaping and reorganization of thought, in harmony with the ideas of critical mathematics education, as understood by Skovsmose and Borba (2004). Thus, Domingues (2020) proposed a triangular representation (Fig. 3b) to illustrate that critical research starts from a situation that does not yet exist, which they call the current situation (CS). To modify a given current situation, an idealized situation (IS) is created by a process called pedagogical imagination (PI). However, not everything that is planned comes to fruition, so the current situation is transformed by means of adaptations from a process called practical organization (PO), which generates the arranged situation (AS), and which is then analyzed by a process called exploratory reasoning (ER). In light of this theory, Domingues (2020) analyzed each of these processes and situations as a human-with-media activity system.

Thus, Domingues (2020) discussed the current situation (before the first festival) as being encapsulated classes, the imagined situation as being the idealized first festival, and the arranged situation as being the realized first festival. There were intershaping relationships between the imagined, current, and arranged systems so that the Festival of Videos could occur. These tensions and adaptations suggest that the transformations occurred in moments of pedagogical imagination, practical organization, and exploratory reasoning of a thinking collective consisting of human and nonhuman actors. Aspects of multimodality were also observed, indicating traces of the technological actors' power of action. This research then suggests a dynamic of the humans-with-media construct with pedagogical transformation. Media are actors in both enabling and disabling a given pedagogical practice and the production of knowledge in a given educational environment, which in this case is the video festival and mathematics education.

When networks of systems are formed, the multiplicity of voices gains an exponential dimension, putting in greater evidence of social and cultural beliefs and values. Without using the idea of networks but expanding this dialogue with activity theory, more specifically regarding multivocality, Canedo Jr (2021) added social semiotics. This leads to a reinterpretation of the principle of multivocality: the "voice" of a digital video is expressed not only with orality but also with the distinct semiotic modes used (gestures, facial features, music, writing, sounds, and images). This theoretical contribution assumes that the epistemological view that permeates the humans-with-media construct has led to the understanding that digital technologies used in video production transform the way the multiple voices that "echo" are heard, are understood, and produce meanings. These results bring us once again to the concept of the agency of digital technologies that, as we have presented throughout this chapter, is identified in various studies that are grounded in humans-with-media.

Between the fourth and fifth phases of digital technologies, the claim that these technologies have agency gains robustness. Studies such as Souto (2013), Galleguillos (2016), Cunha (2018), Domingues (2020), Canedo Jr (2021), and others cited throughout this chapter reinforce this idea. Although for many it may initially appear strange to attribute this characteristic to technologies, in the literature it is possible to verify theorizations that corroborate this idea.

The notion of agency of things, or of technologies, has been present since before the crystallization of the humans-with-media construct, but it has become more explicit in recent years. Canedo Jr (2021) discussed how videos have agency in a thinking collective. Cunha (2020) and Cunha et al. (2022) seek to add a "new layer" to the concept of agency in the humans-with-media construct. Preliminary data from this research suggested a massive manifestation of agency in human-with-media collectives. To analyze the distinct ways in which agency manifests in these collectives, in addition to the humans-with-media construct, Borba (2021) proposed the idea of "fuzzy agency." This research interacts with authors such as Kaptelinin and Nardi (2006), and concepts such as material agency (Malafouris 2008), transformative agency (Sannino 2015; Engeström and Sannino 2020) and collective agency (Haapasaari et al. 2016).

In this same focus on the influence of nonhuman actors, Latour (2001) discussed the active role of microorganisms and microbes in mutations leading to episodes of influenza viruses or HIV. Borba (2021) described a parallel between the agency of media and that of SARS-CoV-2 in the production of knowledge: both change the way we know.

Humans-with-Media and Research in Mathematics Education

Several studies developed during the five phases of digital technologies have been presented in this chapter thus far. They have helped to define the humans-with-media concept more explicitly. In this section, we present empirical studies that exemplify the use of this construct. The general notion of humans-with-media may be exemplified by humans-with-software-Internet or humans-with-videos depending on a given case or study. In this sense, humans-with-media becomes a theoretical lens that helps us understand the interaction between humans and technology. The construct, in turn, is transformed by such studies. Therefore, the construct has been under constant change, even before it had the stable denomination of humans-with-media. This notion is the backbone of the way we understand how knowledge is produced and how technology and humans embody each other.

Humans-with-Software-Internet

The use of devices such as dynamic geometry software and graphing calculators, which characterized the second and third phases of digital technologies in mathematics education, has become the object of investigation in several countries. In this section, we will present research from both phases, as well as examples of the fourth and fifth phases.

In Italy, Arzarello and Robutti (2010) analyzed the multimodal production of ninth and tenth grade students (first and second year of high school) when experimenting with TI-Nspire, a dynamic representation software implemented on calculators and computers, and TI-Navigator, which enables wireless communication between graphing calculators. In this study, two teaching experiments were conducted. In the first, students from one class were organized in pairs and explored the TI-Nspire environment. In the second, another class was organized into small groups and experimented with graphing calculators (TI-84) connected to the TI-Navigator. The data produced in this research were also analyzed under the theoretical lenses of the humans-with-media construct and the concepts of instrumentation and multimodality. According to these authors, the results indicated that the multirepresentation infrastructure of TI-Nspire and the communication infrastructure supported by TI-Navigator supported the students' multimodal production, interaction, and communication in the construction of mathematical meanings.

In Portugal, Jacinto and Carreira (2017) investigated the use of the dynamic geometry software GeoGebra. This research aimed to gain insight into the ways students put their mathematical skills and technological knowledge into action while solving and expressing problems. The context of the investigation was an online competition held outside of school with seventh and eighth grade students. The authors investigated the case of a student who used GeoGebra to solve mathematical problems. The data analysis of this research was supported by three theoretical contributions: the human-media construct, the digital technology problem-solving framework, and techno-mathematical fluency. The case pointed to evidence of techno-mathematical fluency as a combination of thinking mathematically and perceiving technological affordances in a math problem-solving activity beyond school.

The research described above suggests that digital technologies acted in the production of mathematical knowledge, either by supporting this process (Arzarello and Robutti 2010) or by enhancing the ability to solve mathematical problems (Jacinto and Carreira 2017). Studies developed in Brazil, described by Souto (2013) and Galleguillos (2016), indicate the possibility of a greater role of media in the production of mathematical knowledge.

Fast Internet, a characteristic advent of the fourth phase of digital technologies, had agency in the research of Souto (2013) and Galleguillos (2016), as they both developed field work in an online mathematics education university outreach course. The subjects were teachers from various regions of Brazil and other countries who were participants in these courses. Both studies were supported by activity theory and the humans-with-media construct. Souto (2013) focused on the expansive transformations that occur in these collectives, while Galleguillos (2016) analyzed how mathematical modeling processes occur from the perspective of activity theory.

Among the results of this research, Souto (2013) demonstrated the agency of media as a capacity to mobilize expansive transformations by causing the expansion of the object, motives, or artifacts in the analyzed activity systems. Galleguillos (2016), on the other hand, indicated the agency of digital technologies (GeoGebra and Internet) as a capacity to provoke reorganizations of thought in modeling processes.

The results of the research described thus far highlight the ideas emphasized in the theoretical construct systematized by Borba and Villarreal (2005), where the media are not neutral; they have agency and provoke transformations and reorganizations of thinking. Therefore, they influence the production of mathematical knowledge.

Research with humans-with-software-Internet collectives has made important contributions. Souto (2013), Souto and Borba (2016, 2018), and Borba et al. (2022) add another layer to the humans-with-media construct by proposing an analytical perspective: the mini-cyclones of expansive transformations (or learning). These authors reread the expansive cycles proposed by Engeström and Sannino (2009), which are described in predefined steps and generally used for longer periods of time.

The mini-cyclones of expansive transformations were the metaphor used to relate to the atmospheric phenomenon of cyclones. The origin of this concept is explained by Souto (2013) as follows:

> A cyclone is a storm produced by large masses of air animated with great rotational speed and moving at increasing translational speeds. When I propose to come close to this definition I am referring to the movements of this phenomenon, because, besides rotating (moving around itself) and translating (moving around other systems of activity), it is not possible, a priori, to determine or predict exactly the direction it will take. Moreover, I consider it appropriate to relate the idea of an increasing speed to the process of knowledge production and its transformations. (p. 220)

With this approach, it was possible to substitute more random rotational and translational movements in place of the stages foreseen in the expansive cycles. Its beginning, in general, is marked by restlessness and the search for different ways to break a stable pattern of mathematical production that, most of the time, is provoked by the affordances of media. In its development, rotations and translations occur. Rotations correspond to movements that occur within the activity system itself, such as script breaks, experimentations, simulations, conjecture analysis, and reciprocal shaping. Translations refer to the observation of external influences that break patterns and destabilize reproductive beliefs or "encapsulated" practices already ingrained in the production of mathematical knowledge. The possibilities and constraints of media in both rotations and translations define the intensity and direction of these movements. The mini-cyclones reach their end when there is an elaboration of justifications for a given solution produced with different media, which results in new forms by which to express mathematical thinking for a given collective of humans-with-media. Mini-cyclones can contribute to the understanding of how the learning process occurs in a collective of humans-with-media seen as an activity system.

The empirical research with digital technologies (software and Internet) that marked the second and third phases brought important contributions to the development of the theoretical construct discussed in this chapter. Similarly, there are contributions in the fourth phase, with the presence of digital videos, and in the fifth phase, with the diffusion and omnipresence of this technology's possibilities, which occurred due to the advent of fast Internet and mobile devices and the agency of SARS-CoV-2.

Humans-with-Digital-Videos

The fourth phase of digital technologies in mathematics education was marked by the presence of several technologies in classrooms. In this respect, we highlight that fast Internet and smartphones have boosted the production and propagated greater dissemination of digital videos so that they have become part of people's habits and everyday life. Recently, the power of SARS-CoV-2, a nontechnological thing, on the one hand, at some moments, "stopped the planet," and, on the other, intensified the use of digital technologies, signaling the beginning of the fifth phase of technologies in mathematics education (Borba 2021; Borba et al. 2022).

In the remote teaching model in effect during the COVID-19 pandemic, video lectures and web conference recordings were massively used. In addition, there are reports that students frequently resorted to videos with educational content available on platforms such as YouTube.

However, even before the pandemic, the presence of digital videos in classrooms had already awakened the interest of several teachers, researchers, and research groups interested in technologies in education, such as the Research Group on Informatics, Other Media and Mathematics Education (GPIMEM), the "cradle" of the humans-with-media construct. As Borba et al. (2014) detail, the first research contact with videos in GPIMEM began in 2006 in a project in partnership with Canada, which investigated productions with an artistic lens called digital mathematics performance (DMP). Since then, GPIMEM has embraced research such as Domingues (2014), who discussed the use and production of videos in mathematics classes.

The presence of digital videos in mathematics classrooms motivated GPIMEM to develop a larger project in 2015, called E-Licm@t-Tube, and, from 2017, to annually promote the Festival of Digital Videos and Mathematics Education (https://www.festivalvideomat.com), an event that is discussed with an artistic and aesthetic bias in work by Borba et al. (2021) and Domingues and Borba (2021). This project and the festival became the source of several research works (e.g., Oechsler 2018; Neves 2020; Souza 2021; Oechsler and Borba 2020; Canedo Jr 2021), which aimed to deepen our understanding of the participation of media (digital videos) in the production of mathematical knowledge through the lens of the humans-with-media construct and other theoretical foundations. It seemed evident that constructs such as multiple representations (Borba 1993) and humans-with-media needed new allies to transform and be transformed by the notion of videos.

The humans-with-media construct in dialogue with social semiotic theory and the concepts of multimodality enabled Oechsler (2018) to highlight that the modes present in the videos potentiated multimodal communication. In relation to these modes, the author highlighted elements of the mother tongue (orality and writing), mathematical language (visual representation), and cinematographic language (gestures, audio, moving image, costumes, and scenery). Regarding the choices of semiotic modes made by students when producing videos, Neves (2020) said:

> [...] I visualize the theoretical construct human-with-digital videos as the dynamic collective that produces knowledge in qualitatively different ways, considering the semiotic choices made for the intersemioses in the video as a multisemiotic and multimodal phenomenon. These choices conditioned the semantic expansions, creating possibilities for the transformation of mathematical knowledge. (p. 277)

Collectives of humans-with-videos have been gaining more and more space in mathematics education. Its multimodal possibilities with different combinations of semiotic resources present mathematical concepts and ideas in qualitatively different ways from those that were possible only with the use of paper and pencil, for example. According to Borba et al. (2022), historically, even without using social semiotics, the humans-with-media construct has already discussed the use of different communicative resources and their transformative role in knowledge production. However, social semiotics brought robustness to the construct's theoretical framework by presenting an analytical proposal that provides an understanding of how videos transform the mathematics expressed in them.

Regarding analytical aspects, Souza (2021) used film analysis in conjunction with social semiotics and the notion of humans-with-media that favored the understanding of students' semiotic choices in composing the scenes of the videos they produced. For the author, the combination of digital and nondigital technologies generated transformations in mathematical discourses expressed with videos.

More recently, Canedo Jr (2021) investigated how digital video participates in modeling practices when the problem is represented with this media, which he labeled video-problems. The theoretical contributions of the humans-with-media construct, activity theory, and social semiotics allowed the author to show that the multimodal resources of video-problems have the potential to shape the actions of subjects and transform the practice of modeling into a "game" of questions and answers. Moreover, this transformation occurs in a manner distinct from what might occur with other media. Canedo Jr (2021) expanded the ideas of Souto and Borba (2016, 2018) by proposing that the humans-with-media construct be analyzed as a system of activity from a multimodal perspective. Furthermore, the author proposed a reconceptualization of the multivocality principle of activity theory to "consider the media voice not only from what it communicates, but, also, in terms of the modes involved in that communication" (Canedo Jr 2021, p. 136). He also suggested considering aspects related to the different ways of communicating a mathematical idea (multimodality).

The theories derived from the investigations contributed to the development of the humans-with-media construct. They allowed, for example, the observation of students' semiotic choices (different ways of communicating a mathematical idea) in the production of digital videos and the way their thinking is reorganized based on feedback from the media used. The results of this research suggest, in summary, that the forms of expression and reorganization of mathematical thinking occur through different modal combinations of semiotic resources, namely verbal language, mathematical symbolism, still images, animations, and deictic gestures. Thus, multimodality and semiotic choices reaffirm the idea previously presented that digital technologies like videos have agency (Borba et al. 2022).

Social semiotics has become important, because in addition to graphs, tables, and algebraic representation, it brings a tradition of analyzing images, movements, sounds, and more. There are even authors such as O'Halloran (2011, 2015), who emphasized the specific representation of mathematics when analyzing texts on paper. Social semiotics, coupled with activity theory's notion of multivocality, transforms humans-with-media by highlighting how the multiple voices of a system, a principle of activity theory, meet in a video that is expressed with graphics, images of everyday life, and animations. Thus, the media "video production software," along with the video itself, transforms the mathematical knowledge produced. Dialectically, the video, on a site such as the Digital Video and Mathematics Education Festival (https://www.festivalvideomat.com), becomes an actor in a collective of humans-with-media that produces mathematical knowledge in the classroom and in other environments.

We understand that analyzing digital videos from this perspective allows us to partially explain the fascination that videos produce in this century. On the other hand, the notion of humans-with-media allows us to see the agency of video editors, smartphones, and computers in what has become being human. We are human, which is to be historically and culturally marked. Symbolizing history and culture, artifacts imbued with humanity (Borba and Villarreal 2005; Borba 2021) transform the state of being human.

Agency on Humans-with-Internet-SARS-CoV-2

As suggested throughout this chapter, we sought to show how the humans-with-media construct is a contemporary theoretical perspective that has developed during the phases of digital technologies proposed by Borba et al. (2014, 2016, 2022). Although due to space limitations it is not possible to present all the theorizations that emerged from studies based on this construct throughout its history, in this chapter, we sought to discuss in a subtle way how both are intertwined and how other references such as activity theory and social semiotics have contributed to its consolidation.

In this section, we will take a closer look at the interaction between the notions of agency and humans-with-media, which have gained strength, particularly in the fifth phase of digital technologies. We will also discuss the future paths of the humans-with-media construct and the challenges of doing mathematics education during the COVID-19 pandemic and postpandemic.

In the literature, there are different conceptions about agency (power to act). Authors such as Bandura (1989) conceptualized it as a capacity inherent only to human actors. From a similar perspective, the scholars of the Helsinki School of activity theory separated humans from artifacts and emphasized the agency of humans on artifacts. On the other hand, the notion of agency in the humans-with-media construct, already discussed in the synthesis by Borba and Villarreal (2005), puts this discussion in a different perspective. For these authors, human and non-human beings constitute an indivisible unit, and both can exert agency. Borba (2021) summarized the genesis of this notion:

> The notion that both humans and non-humans have agency is part of an effort to model artifacts – in particular, pieces of software, hardware, and the Internet of Things (i.e., things that are connected to the Internet) – as the historical, social, and cultural factors in the collective that produces knowledge. It stresses a view that knowledge is produced (both from a philosophical and a psychological perspective) by humans-with-artifacts. (p. 391)

The idea that knowledge is produced by a collective of humans-with-artifacts suggests that artifacts are not only mediators; they act as coparticipants in this production. This epistemological view reinforces the argument that the power of action of the human-things collective can be manifested by human actors as well as (nonhuman) artifacts. Metaphorically, the notion of agency in the humans-with-media construct is explained as follows: "Agency, therefore, should not be seen as binary, as either present or absent, but having different levels. I see this notion of agency as a 'fuzzy' one, as in fuzzy mathematics, in which we may have degrees of agency [. . .]" (Borba 2021, p. 391).

In this metaphor, by conceiving of agency as something "fuzzy," Borba (2021) suggested that its manifestation in human-things collectives takes place in a "fuzzy," unclear form. Through this perspective, the notion of agency in this theoretical construct supports a nonhierarchical view in which there is no primacy of human beings, nor of things. Both are mutually constituted, or in the words of Borba et al. (2022), "the media are impregnated with humanity, and human beings, impregnated with technologies" (p. 67). This notion gains robustness with the theorizations of other authors (Kaptelinin and Nardi 2006; Malafouris 2008).

According to Kaptelinin and Nardi (2006), humans and nonhumans can manifest agency in three dimensions: (i) needs-based: action is driven by biological and cultural needs, manifested by living beings and social entities; (ii) delegated: when a nonliving being, such as a machine, performs actions that are the result of intentions delegated by human actors; and (iii) conditional: action by anything or anyone that produces desired or undesired effects. In line with these authors, Malafouris (2008) also advocated a nonanthropocentric view of agency.

> While agency and intentionality may not be properties of things, they are not properties of humans either: they are the properties of material engagement, that is, of the gray zone where brain, body and culture conflate. (p. 22)

It is in this perspective, where agency takes place in the "entanglement" of human-things seen as a total system, that we find the notion of agency associated with the humans-with-media construct (Borba 2021; Souto and Borba 2016, 2018; Borba et al. 2022; Borba and Villarreal 2005). This perspective is at the heart of developing research proposed by Cunha (2020) and Cunha et al. (2022). These authors are investigating how the interrelationships between actors – human (teachers and students), technological (digital videos), and nonliving things (SARS-CoV-2) – result in the manifestation of agency in human-digital-video systems. Thus, it is expected to broaden the notion of agency by deepening our understanding of how this process occurs and the transformations it brings about in these systems.

The idea that nonliving things can manifest agency in human-things collectives has been enhanced by the power of SARS-CoV-2, the cause of the COVID-19 pandemic. In a short period of time, this virus caused deep transformations in society as a whole (Borba 2021). Borba (2021) stated that in mathematics education, a universal effect of this pandemic was "a tendency to 'go online': shop online, meet friends online, and learn online" (p. 386).

Borba et al. (2022) suggested that the power of the SARS-CoV-2 virus marks the fifth phase of digital technologies in mathematics education. The authors argued that this virus has enhanced the development and use of these technologies in several areas. Livestreams, digital videos, and a wide variety of virtual learning environments are some of the artifacts that characterize this phase. The historical moment that marks the beginning of this phase coincides, therefore, with the COVID-19 pandemic.

On the one hand, the power of SARS-CoV-2 intensified the use of digital technologies in mathematics education during the pandemic period. On the other hand, it made social inequality more evident in Brazil and in the world (Borba 2021; Borba et al. 2022). At a time when there was still no vaccine that could control the contamination and exponential transmission of the infection caused by this virus, social isolation measures became the only possible way to avoid a collapse of the health system and save thousands of lives. However, these measures impacted the poorest much more severely. In particular, the workers in informal employment, whose inability to continue work activities resulted in a loss of food on the table for their families. Unlike the richest who were able to isolate themselves in paradise on private islands, and the ultrarich, especially the chief executive officers (CEOs) of companies in the digital sector, such as Facebook, Microsoft, and Tesla, who increased their fortunes by 274% with the appreciation of shares during the health crisis (Centro de Estudos Estratégicos da Fiocruz – 2020).

The social gap between the rich and poor deepened during the COVID-19 pandemic. The Food and Agriculture Organization of the United Nations (FAO) report, prepared in conjunction with other entities, points out that hunger increased dramatically in the world during the pandemic period (FAO et al. 2021). In addition to social factors, the pandemic also aggravated educational inequalities. In Brazil, remote education was not equal for all humans, in particular due to disparities in the media available and home conditions in each collective of humans-with-media.

In front of the interruption of face-to-face classes during the period of social isolation, private schools – with greater availability of resources and structure – quickly returned to their educational activities online. In general, students in these schools had favorable conditions for studying in virtual environments, such as the availability of individual rooms and technologies – fast Internet, computers, smartphones, among others. On the other hand, in many public schools, remote teaching was only possible through the workbook system, in which printed materials were sent to schools and slowly distributed to students. This is because, in Brazil, the access to the Internet is still very unequal, as pointed out by a Brazilian Institute of Geography and Statistics (IBGE) survey that indicated that one in every five Brazilians does not have access to the Internet (Brazilian Internet Association [ABRANET] 2021).

In addition to barriers to virtual education due to the lack of Internet availability, there were those in which the access conditions were precarious. In poorer homes, when there was a single device (computer or cell phone), it was shared with several other children in a cramped room. This precariousness certainly limited the school learning of students from poorer families during the pandemic period. Borba (2021) argued that "trying to solve a mathematics problem in a crowded house in a slum is very different than doing so in a spacious, luxurious apartment with a veranda" (p. 394). Homes, digital technologies, and parents played important roles in student learning during the remote teaching period.

The understanding that the collective includes "things" such as homes leads to an expansion of the humans-with-media concept of media. The interplay of this theoretical construct with activity theory has made it possible to sustain the idea that these nonhuman actors (media) "do not necessarily play the secondary role of learning mediators, but are also protagonists in this process" (Borba et al. 2022, p. 113).

According to Borba (2021), "the notion that both humans and non-humans have agency is part of an effort to model artifacts – in particular, pieces of software, hardware, and the Internet of Things (i.e., things that are connected to the Internet) – as the historical, social, and cultural factors in the collective that produces knowledge" (p. 391). A critique of the rigidity of the triangles of the third generation of activity theory is presented in Souto and Borba (2016, 2018). They propose that agency would be distributed in many vertexes of the triangles that represent an activity system, not only in the subject vertex as proposed by Engestrom (1987) and as presented in Fig. 1. Rather than a rigid structure, agency could permeate all elements of the triangular diagram as the system develops. Thus, we leave for future research the challenge of building a new representation that can break with the supposed "rigidity" of this triangular diagram.

This dialogue has the rhizomatic form in which no propositions or statements are hierarchical or branch out in a more fundamental way than another. In this sense, assuming that media manifests agency in human-things collectives raises questions such as: How does the process of knowledge production occur in human-things collectives? What is the place of media in this process? What transformations occur when nonhuman actors manifest agency in human-with-media collectives? Conversely, can we think that different humans may be located in different places on a spectrum of agency? What place, in the fuzzy spectrum, does media occupy in a given collective of humans-with-media? Can agency also be seen as collective in a collective of humans with media?

We believe that these are emerging research questions, whose elucidation can contribute not only to consolidating the epistemological view of the human-beings-with-media construct but also to broadening our understanding of how the power of nonhuman things to act can transform learning, the classroom, and society.

Final Remarks

> We have collectives of home-parents-internet-student-teacher as the minimal unit of the collective agent who produces knowledge. Home and parents, things and humans, have added more to social inequality and to discussions about how to use digital technology in mathematics education. (Borba 2021, p. 396)

As previously noted, the humans-with-media construct has a history. We associated this history with different phases. The fifth phase, associated with the large use of digital technology during the pandemic, made evident the way that homes also have agency in the production of knowledge. Phases value different technology and in turn different technology have influenced and transformed the construct itself. We believe that the humans-with-media construct can be relevant for the debate regarding artificial intelligence, a debate that seems to emerge, once more, with the agency of technologies such as ChatGPT and a possible new phase.

In this chapter, we have discussed the changes and dynamics of a theoretical construct. Humans-with-media has, as pointed out, roots in phenomenology, in activity theory, and in notions such as multiple representations. However, there are two notions that are paramount and that have not yet been addressed: ethnomathematics and the liberation pedagogy of Freire (Freire et al. 1988). As discussed in the previous section, the notion of humans-with-media was formally extended to humans-with-media-homes in the fifth phase of the use of digital technologies in mathematics education. SARS-CoV-2 brought the advent of such an intensive use of digital technologies, which led to the denomination of a fifth phase. If the use of digital platforms for online communication was prominent in this fifth phase, the same was not true of another nonhuman actor: the different conditions of households. While there are those who enjoyed a single room for their online communication, others had to share a cell phone and a room with siblings and parents while studying and communicating in general.

This is how Borba (2021) argues that there is a resurgence of the critical mathematics education agenda. Issues that should be in the forefront of the agenda include differences in homes, differences in the quality of the Internet, and differences in the help that parents could give their children at home, whether due to previous studies or due to the time they had to dedicate to work. Parental assistance with math was even more problematic, as there is a public image of mathematics (Silva 2020) that has kept mathematics away from parents and children for generations.

Freire's idea of dialogue is used prominently in work by Borba et al. (2022) to illustrate how digital videos in mathematics education are a way to value the culture of the twenty-first century student who frequently communicates through videos. This same idea has been used since the beginning of the design of the humans-with-media construct, either to value the computer media used by some students in the last decade of the 1900s or to value the orality media of illiterate adults in Brazilian favelas (Borba 1988). These adults, through orality, express their mathematics imbued in a medium, in the same way that students show their mathematics with pencil and paper and increasingly with software and videos.

Homes, like libraries, are media and have agency, so the construct can be broadly called humans-with-media, although we have shown several variants of it in this chapter. This construct has an epistemological dimension by emphasizing the role of media in a broad sense, an ontological dimension by emphasizing how media constitute and are constituted by humans, and a critical and political dimension by emphasizing the different possibilities of mathematics being produced by different groups, as D'Ambrósio (1999) proposed. While difference should be cherished, it should also be denounced in regard to asymmetries that prevent mathematics education in homes that experience barriers to an online education, or for a study that complements the one done at school. We need mathematics education for all that celebrates the diversity of humans and equal access to food, home, Internet, and a welcoming environment for education!

Acknowledgment Although she is not responsible for the content, we would like to thank Jessica Shumway for her comments in earlier versions of this chapter. We would like to thank CNPQ, Grant number 309992/2020-6, and FAPEMAT, Grant number 0206965/201, for supporting the research presented in this chapter.

References

Arzarello F, Robutti O (2010) Multimodality in multi-representational environments. ZDM 42(7): 715–731

Bandura A (1989, September) Human agency in social cognitive theory. Am Psychol 44(9): 1175–1184

Borba MC (1987) Um estudo de etnomatemática: sua incorporação na elaboração de uma proposta pedagógica para o "núcelo-escola" da favela da Vila Nogueira – São Quirino [A study of Ethnomathematics: its incorporation in the elaboration of a pedagogical proposal for the "núcelo-escola" of favela da Vila Nogueira – São Quirino]. Masters dissertation in mathematics education – Universidade Estadual Paulista (UNESP), Rio Claro, SP

Borba MC (1988) Etnomatemática: a matemática da favela em uma proposta pedagógica. [Ethnomathematics: favela mathematics in a pedagogical proposal] In: Freire P, Nogueira A, Mazza D (eds) Na escola que fazemos uma reflexão interdisciplinar em educação popular [A school we make: an interdisciplinary reflection on popular education]. Vozes, pp 71–77.

Borba MC (1993) Students understanding of transformations of functions using multi-representational software. Doctoral dissertation, Cornell University, Cornell
Borba MC (1999) Tecnologias informáticas na educação matemática e reorganização do pensamento. [Computer technologies in mathematics education and reorganization of thinking]. In: Bicudo MAV (ed) Pesquisa em educação matemática: concepções e perspectivas [Research in mathematics education: concepts and perspectives]. Editora Unesp, pp 285–295
Borba MC (2012) Humans-with-media and continuing education for mathematics teachers in online environments. ZDM-Math Educ 44(6):801–814. https://doi.org/10.1007/s11858-012-0436-8
Borba MC (2021, April 27) The future of mathematics education since COVID-19: humans-with-media or humans-with-non-living-things. Educ Stud Math:385–400. https://doi.org/10.1007/s10649-021-10043-2
Borba MC, Villarreal ME (2005) Humans-with-media and the reorganization of mathematical thinking: information and communication technologies, modeling, visualization and experimentation. Springer US
Borba MC, Silva RSR, Gadanidis G (2014) Fases das tecnologias digitais em educação matemática: sala de aula e internet em movimento [Phases of digital technologies in mathematics education: the classroom and the internet in motion], 1st edn. Autêntica
Borba MC, Domingues NS, Lacerda HDG (2015) As tecnologias audiovisuais em educação matemática investigadas no GPIMEM [Audiovisual technologies within mathematics education in GPIMEM]. In: Sant'ana CC, Santana IP, Amaral RS (eds) Grupo de estudos em educação matemática: ações cooperativas e colaborativas construídas por várias vozes [Study groups in mathematics education: cooperative and collaborative actions constructed by many voices], 1st edn. Pedro & João Editores, pp 285–312
Borba MC, Askar P, Engelbrecht J, Gadanidis G, Llinares S, Aguilar MS (2016) Blended learning, e-learning and mobile learning in mathematics education. ZDM-Math Educ:589–610
Borba MC, Domingues NS, Costa RF (2021) O Festival de vídeos digitais e educação matemática: um olhar para as experiências estéticas. [The digital videos and mathematics education festival: a look at aesthetic experiences] In: Scucuglia R, Idem RC (eds) Experiências estéticas em educação matemática. [Aesthetic experiences in mathematics education]. (1st ed. pp. 234–271). Editora Fi
Borba MC, Souto DLP, Canedo NR Jr (2022) Vídeos na educação matemática: Paulo Freire e a quinta fase das tecnologias digitais [Videos and mathematics education: Paulo Freire and the fifth phase of digital technology]. Autêntica
Brazilian Internet Association (2021, April 14) IBGE: 40 milhões de brasileiros não têm acesso à internet [IBGE: 40 million Brazilians do not have access to the internet]. Abranet. Retrieved June 27, 2022, from https://www.abranet.org.br/Noticias/IBGE%3A-40-milhoes-de-brasileiros-nao-tem-acesso-a-Internet-3345.html?UserActiveTemplate=site#.Yrl0hXbMLIU
Canedo Jr NR (2021) A participação do vídeo digital nas práticas de modelagem quando o problema é proposto com essa mídia [The participation of digital vídeos in modeling practices when a problem is in a vídeo medium format]. Doctoral dissertation in mathematics education – Universidade Estadual Paulista (UNESP), Rio Claro, SP
CEE-Fiocruz (2020, October 5) Por que ricos ficaram mais ricos e pobreza explodiu na pandemia? CEE Fiocruz. [Why did the rich get richer and poverty explode in the pandemic? CEE Fiocruz]. Retrieved June 24, 2022, from https://cee.fiocruz.br/?q=por-que-ricos-ficaram-mais-ricos-e-pobreza-explodiu-na-pandemia
Cunha JFT (2018) Blended learning e multimodalidade na formação continuada de professores para o ensino de Matemática. [Blended learning and multimodality in the continuing education of mathematics teachers]. Masters dissertation in teaching science and mathematics – Universidade do Estado de Mato Grosso (UNEMAT), Barra do Bugres, MT
Cunha JFT (2020) Licenciatura híbrida em matemática: qual o papel dos vídeos digitais? Anais do XXIV Encontro Brasileiro de Estudantes de Pós-Graduação em Educação Matemática
Cunha JFT, Borba MC, Souto DLP (2022) Agency em coletivos seres-humanos-com-vídeos: escolhas teóricas e metodológicas de uma investigação na pandemia da COVID-19 [Agency in human-with-videos collectives: theoretical and methodological choices of an inquiry in the COVID-19 pandemic]. Anais do XIV Encontro Nacional de Educação Matemática.

D'Ambrósio U (1999) A história da Matemática: questões historiográficas e políticas e reflexos na educação matemática. [The history of mathematics: historiographical and political issues and reflections on mathematics education]. In: Bicudo MAV (ed) Pesquisa em educação matemática: concepções e perspectivas [Research in mathematics education: concepts and perspectives]. Editora Unesp, pp 97–115
Domingues NS (2014) O papel do vídeo nas aulas multimodais de Matemática Aplicada: uma análise do ponto de vista dos alunos [The role of video in multimodal Applied Mathematics classes: an analysis from the students' point of view]. Masters dissertation in mathematics education – Universidade Estadual Paulista (UNESP), Rio Claro, SP
Domingues NS (2020) Festival de vídeos digitais e educação Matemática: uma complexa rede de sistemas seres-humanos-com-mídias. [Digital videos festival and mathematics education: a complex network of systems of humans-with-media]. Doctoral dissertation in mathematics education – Universidade Estadual Paulista (UNESP), Rio Claro, SP
Domingues NS, Borba MC (2021) Digital video festivals and mathematics: changes in the classroom of the 21st century. Journal of Educational Research in Mathematics 31(3):257–275
Engeström Y (1987) Learning by expanding: an activity-theoretical approach to developmental research. Orienta-Konsultit
Engeström Y (1999) Activity theory and individual and social transformation. Perspect Act Theory 19(38):19–30
Engeström Y (2001) Expansive learning at work: toward an activity theoretical reconceptualization. J Educ Work 14(1):133–156
Engeström Y, Sannino A (2009) Studies of expansive learning: foundations, findings and future challenges. Educ Res Rev 5(1):1–24
Engeström Y, Sannino A (2020) Toward a Vygotskian perspective on transformative agency for social change. In: Revisiting Vygotsky for social change: bringing together theory and practice, pp 87–109
FAO, IFAD, UNICEF, WFP, & WHO (2021) In brief to the state of food security and nutrition in the world 2021. Food and Agriculture Organization of the United Nations. Retrieved June 24, 2022, from https://doi.org/10.4060/cb5409en
Frankenstein M (1987) Critical mathematics education: an application of Paulo Freire's epistemology. In: Shor I (ed) Freire for the classroom. Boynton/Cook, Portsmouth, pp 180–210
Freire P, Nogueira A, Mazza D (1988) Na escola que fazemos uma reflexão interdisciplinar em educação popular. [A school we make: an interdisciplinary reflection on popular education]. Vozes
Galleguillos JEB (2016) Modelagem matemática na modalidade online: análise segundo a Teoria da Atividade [Mathematical modeling in the online modality: analysis according to the Activity Theory]. Doctoral dissertation in mathematics education – Universidade Estadual Paulista (UNESP), Rio Claro, SP
Haapasaari A, Engeström Y, Kerosuo H (2016) The emergence of learners' transformative agency in a change laboratory intervention. J Educ Work 29(2):232–262
Halliday MAK (1993) Towards a language-based theory of learning. Linguist Educ 5(2):93–116. https://doi.org/10.1016/0898-5898(93)90026-7
Jacinto H, Carreira S (2017) Mathematical problem solving with technology: the techno-mathematical fluency of a student-with-GeoGebra. Int J Sci Math Educ 15(6):1115–1136
Kaptelinin V, Nardi BA (2006) Acting with technology: activity theory and interaction design. MIT Press
Latour B (2001) A esperança de Pandora [Pandora's Hope] (Sousa GCC, trans). EDUSC
Latour B (2020, March 26) Is this a dress rehearsal? In the Moment. https://critinq.wordpress.com/2020/03/26/is-this-a-dress-rehearsal/
Lave J (1988) Cognition in practice: mind, mathematics and culture in everyday life. Cambridge University Press
Lévy P (1993) As tecnologias da inteligência: O futuro do pensamento na era da informática. [The intelligence technologies: the future of thinking in the information era], 1st edn. EDITORA 34

Malafouris L (2008) At the potter's wheel: an argument for material agency. In: Knappett C, Malafouris L (eds) Material agency: towards a non-anthropocentric approach. Springer, pp 19–36

Neves LX (2020) Intersemioses em vídeos produzidos por licenciandos em Matemática da UAB. [Intersemiosis in videos produced by UAB mathematics undergraduates]. Doctoral dissertation in mathematics education – Universidade Estadual Paulista (UNESP), Rio Claro, SP

O'Halloran KL (2011) Historical changes in the semiotic landscape: from calculation to computation. In: Jewitt C (ed) The Routledge handbook of multimodal analysis. Routledge, pp 98–113

O'Halloran KL (2015) The language of learning mathematics: a multimodal perspective. J Math Behav 40:63–74

Oechsler V (2018) Comunicação multimodal: produção de vídeos em aulas de Matemática. [Multimodal communication: video production in mathematics classes]. Doctoral dissertation in mathematics education – Universidade Estadual Paulista (UNESP), Rio Claro, SP

Oechsler V, Borba MC (2020) Mathematical videos, social semiotics and the changing classroom. ZDM 52(5):989–1001. https://doi.org/10.1007/s11858-020-01131-3

Powell AB, Frankenstein M (1997) Ethnomathematical knowledge. In: Powell AB, Frankenstein M (eds) Ethnomathematics: challenging eurocentrism in mathematics education. SUNY Press, pp 5–13

Sannino A (2015) The emergence of transformative agency and double stimulation: activity-based studies in the Vygotskian tradition. Learn Cult Soc Interact 4:1–3

Santa Ramirez ZM (2016) Producción de conocimiento geométrico escolar en un colectivo de professores-com-doblado-de-papel. [Production of school geometric knowledge in a collective of teachers-with-folded-paper]. Doctoral dissertation – Universidade de Antioquia, Antioquia (CO)

Schütz A (1970) On phenomenology and social relations: selected writings (Wagner HR, ed). University of Chicago Press

Silva RSR (2020) On music production in mathematics teacher education as an aesthetic experience. ZDM 52(5):973–987

Skovsmose O, Borba MC (2004) Research methodology and critical mathematics education. In: Valero P, Zevenbergen R (eds) Researching the socio-political dimensions of mathematics education: issues of power in theory and methodology. Kluwer, Dordrecht, pp 207–226. https://doi.org/10.1007/1-4020-7914-1_17

Souza MB (2021) Vídeos digitais produzidos por licenciandos em matemática a distância. [Digital videos produced by distance learning mathematics graduates]. Doctoral dissertation in Mathematics Education – Universidade Estadual Paulista (UNESP), Rio Claro, SP

Souto DLP (2013) Transformações expansivas em um curso de educação matemática a distância online. [Expansive transformations in an online distance mathematics education course]. Doctoral dissertation in mathematics education – Universidade Estadual Paulista (UNESP), Rio Claro, SP

Souto DLP, Araújo JL (2013) Possibilidades expansivas do sistema seres-humanoscom-mídias: um encontro com a teoria da atividade. [Expansive possibilities of the human beings-with-media system: an encounter with activity theory]. In: Borba MC, Chiari AS (eds) Tecnologias digitais e educação matemática. [Digital technologies and mathematics education]. Livraria da Física

Souto DLP, Borba MC (2016) Seres humanos-com-internet ou internet-com-seres humanos: uma troca de papéis? [Humans-with-internet or internet-with-humans: an exchange of roles?]. Rev Latinoam Investig Mat Educ 19(2):217–242. https://doi.org/10.12802/relime.13.1924

Souto DLP, Borba MC (2018) Humans-with-internet or internet-with-humans: a role reversal? (Reprint). RIPEM 8(3):2–23

Tikhomirov OK (1981) The psychological consequences of computarization. In: Wertsch JV (ed) The concept of activity in soviet psychology. M. E. Sharpe, pp 256–278

Wagner HR (1979) Fenomenologia e relações sociais: textos escolhidos de Alfred Schütz [Alfred Schütz on phenomenology and social relations] (Melin Â, trans). Zahar

Zhao R, Walke B, Hiertz GR (2006) An efficient IEEE 802.11 ESS mesh network supporting quality-of-service. IEEE J Select Areas Commun 24(11):2005–2017