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ORIGINAL ARTICLE

Mathematical videos, social semiotics and the changing classroom

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Abstract

With the advancement of digital technology, the roles of teachers and students are slowly changing. The classroom is on the verge of becoming a new, more open place, one with fewer barriers to the rest of society. Our purpose in this paper is to discuss how the creation of videos with mathematical content may contribute to this process of rendering the classroom borderless, as well as how this activity can become a teaching and learning tool. We ground our discussion in social semiotics, a theory that considers the context of production and the negotiations between actors to analyze the meanings produced. We report on the production of videos by middle school students and the steps involved: discussion, editing and dissemination. At each step, data were produced and collected. We focus our analysis on the question of how the production of videos in the classroom can help in the communication of mathematical knowledge and in the change of the dynamics of the classroom. We find that video production provides a classroom dynamic in which students can become protagonists in the teaching and learning process, with teachers mediating this activity. We argue that video production is a different way to express mathematics, and it is particularly well-suited to expressing what students have understood. Using videos, a new kind of mathematics can emerge in the classroom, joining its traditional symbolic language with other modes, such as language, gesture, image and music.

Keywords Social semiotics · Humans-with-media · Sign of learning · Video production · Internet · Classroom changes

1 Introduction

> Student: Teacher, could you explain how I can calculate the chances of winning in a lottery game?
> Teacher: That has to do with combination. In the last class we only explored the fundamental principle of counting, remember?
> Student: I missed the class, but I searched for videos to help me understand what you explained, and I was not sure about this example.

This dialogue took place between one of the authors of this article and one of her students while explaining the subject of combinatorial analysis in high school. Similar behavior—searching for explanations in other media, with other teachers—is becoming more and more common among students in classrooms around the world. What is beginning to happen in some classrooms is a type of blended learning in which video is used as a 'backup' for the teacher's explanation of a topic that was missed by a student, as illustrated by the excerpt above: from the student's question, the teacher could once more review concepts related to combinatorial analysis and reflect on the student's doubt along with all the other students. Some previous research has described and incentivized this kind of blend of online and face-to-face classroom studies (Borba et al. 2016; LaFee 2013; Owen and Dunham 2015). Borba et al. (2016) cited several uses of technology in the classroom that generate blended learning. However, in these papers, when videos are discussed, the emphasis is on the use of videos already found on the Internet. In this paper, we explore the process of video production by students and teachers.

The dialogue presented above is just one example of the many lived by the authors of this article in the classroom, and also found in research reports such as that of Domingues

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(2014). There is an increasing number of students who look for videos with mathematical content on platforms such as YouTube to answer questions generated in the classroom or, in the case of online education, to supplement the lack of a physical teacher (Borba et al. 2018). These cases illustrate how the classroom is transforming with the integration of the Internet. Content is no longer presented in a strict linear order, because the questions of the students transcend the textbook and its organization, making the process of teaching and learning more dynamic and student-centred. In the case explored above, if a strict linearity were followed, the teacher would explore the fundamental principles of counting, permutation, arrangement and combination, in that order. However, the student's doubt reversed the order of presentation, and was used as a catalyst to discuss examples and concepts of combinatorial analysis.

We can see this as an emergent facet of education, in which the student searches for content outside the classroom, usually through videos, and brings his or her doubts about and understandings of these videos into the classroom, reflecting with their teacher and colleagues about the content. This recalls Freire (1970) idea of a dialogical education, in which students learn from teachers, but teachers also learn from students. Students are 'native' in digital technology and may combine digital videos and mathematics education in ways teachers—usually older, non-native or 'not as native'—may not think of. With Freire's notion of dialogical education on one hand, and the changes provided by digital technology, particularly regarding digital videos, on the other, in the last few years our research group has developed a research project driven by questions such as the following: in an education in which the student has a protagonist role, why not allow them to produce their own videos? Is there a change happening in mathematics that is expressed through videos? With this activity, is there a change in the dynamics of the classroom?

We can separate the use of digital technology in the mathematics classroom into four phases (Borba et al. 2016). The first phase was characterized by the use of LOGO software and of programming. The second phase was marked by the creation of computer labs in schools and the use of software programs which allowed visualization and experimentation with mathematical demonstrations. The third phase began with the creation of the Internet, which made it possible easily to disseminate online courses. The fourth phase began with the advent of high-speed Internet and is characterized by the use of videos, dynamic software, and collaboration activities, among others.

It is this fourth phase in which education has been situated for the past few years: most students have access to a variety of information, and can view, experiment with, and conjecture about information that could only be read and accepted before. Now, they can create their own videos and tools to explore subjects that interest them. Despite these developments, production of videos with mathematical content by students themselves is not a very frequent practice in the classroom, although some research projects have encouraged student video production, such as the project 'Digital Videos in Distance Learning Mathematics', coordinated by one of the authors of this paper and financed by Brazil's National Council for Scientific and Technological Development (CNPq). Several research studies have originated in this project, with the production of videos by students in basic education (middle and high) schools (Oechsler 2018; Oechsler and Borba 2018; Oechsler and Borba 2017; Oliveira 2018), as well as in undergraduate courses in mathematics, in both face-to-face (Souza 2017) and distance learning (Fontes 2019; Neves and Borba 2017; Silva 2018).

Borba and Oechsler (2018) conducted a survey on the use of videos in the classroom. In their analysis, they separate the use of videos in the classroom into three types: (i) recording lessons for later analysis of the teaching and learning process; (ii) using videos available on educational platforms or social media as didactic material; (iii) video production by students and teachers in the classroom. Type (i) is very commonly used in academic research, to analyze teachers' practices and students' learning processes. Much of this research is developed outside Brazil (see, e.g., Barlow et al. 2014; Coles 2015; Huang and Li 2009; Jaworski and Huang 2014; Johansson et al. 2014; Rowland et al. 2014).

Types (ii) and (iii) have educational objectives, rather than research objectives, although there are different types of research associated with them. In modality (ii), students use videos available in social media and educational platforms to answer questions, review content, and study content lost in the classroom (Kolikant and Broza 2011; Saxe et al. 2013). These videos are usually produced by teachers (who are not the teachers of the students accessing them) and posted on educational platforms or social media channels. Often, they are teachers who produce these channels full-time. This can be seen as the external environment being introduced into the classroom.

The activities reported in this paper fall under type (iii), which explores the production of videos, both by students and by teachers. The focus of this paper is on the production of videos with mathematical content by students in the classroom. Many students already produce and share videos for the purpose of entertainment. This research expands this immanent activity to videos with a mathematics education perspective.

We discuss research results focusing on the following research question: How can video production become a teaching and learning tool for the students? Based on Social Semiotics, we analyze how the production of videos in the classroom can help in the communication of mathematical knowledge and in the change of the dynamics of the classroom.

2 Social semiotics and humans-with-media: a discussion of video and knowledge production

According to Borba and Oechsler (2018), student video production in mathematics classes is not a common practice. Nonetheless, it has been growing in recent years, such as with video production festivals like the Math Performance Festival in Canada (Borba et al. 2014) and the Festival of Digital Videos and Mathematical Education in Brazil (Domingues and Borba 2018).

Because student video production in mathematics class is not yet an established practice, there is no standardized methodology for classifying and analyzing student produced video. Festivals typically use criteria such as the nature of the mathematical idea, creativity and imagination and artistic-technological quality (Domingues and Borba 2018) to analyze students' videos. These criteria help in the evaluation of the video. But how can we tell if the production of video helped in communicating the mathematical knowledge of those involved? What negotiations between the actors resulted in a reorganization and production of mathematical knowledge? With the final video alone, we cannot answer these questions. To this end, we seek to anchor our research on the process of video production and analysis in social semiotics. Under the theory of social semiotics, in order to analyze a given sign, the context must be understood as well.

> A primary focus of social semiotic multimodal analysis is on mapping how modal resources are used by people in a given community/social context, in other words sign-making as a social process. The emphasis is on the sign-maker and their situated use of modal resources. This foregrounds the question of what choices people make (from the resources available to them) and the non-arbitrary and motivated character of the relationship between language and social context. There is therefore a strong emphasis on the notion of context within social semiotic multimodal analysis. The context shapes the resources available for meaning-making and how these are selected and designed. (Jewitt 2009, p. 30).

Under Social Semiotics, by knowing the context of production, it is possible to understand the choices made by the sign-maker, and how these choices influenced the reorganization of his or her thinking and the production of meaning. Social semiotics is based on the ideas of the linguist Michael Halliday, who gives a critical reading of the works of Saussure, Peirce and Voloshinov. Traditional or "pure" semiotics as explored by these authors "emphasizes structures and codes, at the expense of functions and social uses of semiotic systems, the complex interrelations of semiotic systems in social practice, all of the factors which provide their motivation, their origins and destinations, their form and substance." (Hodge and Kress 1988, p. 1). In the framework of social semiotics, though, meanings are not the same for all users, and they must be studied at the level of social action, along with their effects in the production of meaning (Hodge and Kress 1988).

> Social semiotics is an attempt to describe and understand how people produce and communicate meaning in specific social settings, be they settings such as the family or settings in which sign-making is well institutionalized and hemmed in by habits, conventions and rules. (Kress and Van Leeuwen 2006, p. 266).

Social Semiotics has two central categories: sign and mode. Signs are elements by which people interpret and express meaning for communicative purposes. An example might be the color red, which is a sign in many cultures for danger. Modes are means of making representations of elements (sounds, images, among others). Some potential modes include image, sound, speech, gesture, writing. In the production of videos specifically, some modes are more characteristic, such as frames, camera position, sound and particularly moving picture, which is the characteristic mode of cinematographic language (Aumont 1992; Metz 1991). Because social semiotics concerns not only the sign itself, but also the process of producing its meaning, we adopt this theory in the data analysis in order to understand the process of production and reorganization of thinking during video production. This involves analyzing not only the mathematical signs and the design presented in the videos, but also the negotiation process between the sign-makers in the construction of these signs: "In a Social Semiotic theory, signs are *made*—not *used*—by a sign-maker who brings meaning into an *apt* conjunction with a form, a selection/choice shaped by the sign-maker's *interest*" (Kress 2010, p. 62).

The video production analyzed in this paper has mathematical content as its central theme. Just like video, mathematics has its own characteristic modes: language, symbolism and visual representation (O'Halloran 2000, 2005). According to the interest of the sign-maker, several of these modes can be used together; this is known as multimodality (Bezemer and Kress 2016; Jewitt 2009). The goal of multimodality in the present context is to assist in the presentation and learning of mathematical concepts.

In mathematics, the language mode refers to both speech and writing. This mode is often used to present, contextualize and describe a mathematical problem. In order to communicate the problem, the speaker needs to organize their mathematical ideas through their language, whether oral or written.

Symbolism, another characteristic mode of mathematics, has as one of its functions the translation into a symbolic language of what was initially expressed in oral, written or visual language: “The underlying premise is that mathematical symbolism developed as a semiotic resource with a grammar which had the capacity to solve problems in a manner that is not possible with other semiotic resources” (O’Halloran 2005, p. 15).

Once the problem has been explained, it is possible to create a visual representation of the ideas, such as in the form of graphs or diagrams. Such a visualization has the goal of helping in the organization of ideas that were initially explained in language or in symbolism.

We agree with O’Halloran (2000) that mathematics is multimodal “because the linguistic, visual and symbolic semiotic systems differentially contribute to the meaning of the text” (p. 300). In our case, the combination of mathematics multimodality and the various video modes multiplies the modes’ potential (Lemke 1998), contributing to the communication of mathematical content and to the reorganization of the signmaker’s thinking. The function of social semiotics is to describe the potentials of each mode and how these potentials can be multiplied when used together.

It is important to consider that signs are produced (according to the interest of the sign-maker) in a collective of actors. Borba and Villarreal (2005) regard knowledge as something produced by a collective of human and non-human actors, called humans-with-media, in which each actor plays a central role. Thinking about education situated in the fourth phase of technological development, the collective of human actors is composed of the students, teacher, family, society, and other humans that can participate in students’ discussions and reflections. Non-human actors are numerous and varied, from material technologies (such as computers, cell phones, cameras, among other instruments and tools) to immaterial technologies (orality, writing, computer science, thought) (Borba 1999).

The collective of humans-with-media, through interaction, negotiation, and reflection, is able to produce knowledge. From this perspective, “humans are constituted by technologies that transform and modify their reasoning and, at the same time, these humans are constantly transforming these technologies” (Borba and Villarreal 2005, p. 22). This is the notion of the *intershaping relationship*, which is one of the main notions associated with humans-with-media as a theoretical construct: human beings and media influence and shape each other, contributing to the reorganization of thought and the production of new practices and knowledge (Borba 2012). According to Borba and Villarreal (2005) and Borba (2012), the collective of humans-with-media should be considered as the minimum unit of analysis. There can be no production of human knowledge without the influence of media, nor can any media be developed without the influence of humans. When humans interact with media, they reorganize their thinking according to its affordances (Souto and Borba 2016).

Research conducted by the Informatics, Other Media and Mathematics Education research group (GPIMEM) has led us to the conclusion that the use of different media will result in different ways of expressing mathematical ideas (Borba 2012). As an example, we can imagine the mathematical language used in an online chat. In this case, one needs to write in language what one wants to explain in symbolic language, because most chat programs still do not permit the use of the symbolic language of mMathematics. For example, the derivative symbol ($\frac{\partial f}{\partial x}$) cannot be inserted directly, and must be spelled out as ‘derivative’. On the other hand, in a video (depending on the medium used to record and the other materials available), we can not only use mathematical symbolic language, but we can also use other modes, such as gestures and facial expressions, beyond mathematical symbolic language. This alters the possibilities of mathematical production (Borba 2012), because we can use new modes to produce meaning. Gestures and moving images can help to explain some content that, in a static way, or in a written mode, was not understood by the student.

In mathematics class, when solving traditional problems, the student’s way of expressing his or her knowledge is often through the use of symbolic language. However, the symbolic language cannot convey the student’s thoughts during the solution, nor what knowledge was necessary to solve the problem. When preparing a video, though, in order to explain the problem-solving process, the student needs to expose these reflections. These negotiations are often perceived during the video production process, as students choose the theme and the approach to be used, and study what they will explore. Following this process allows the teacher to understand students’ difficulties, solutions, and the choices they make in the production of video. By having access to this context of production, it is possible to understand whether or not the students have learned the content, as their negotiations and their reorganizations of thinking are analyzed.

In the video production task, students interact with several actors, both human and non-human. In this interaction, they must reflect on the affordances of each medium used, they must decide what modes they will use, and they must decide what type of video they will produce. Thinking of this interaction as an activity system, we can see that technology is not only an “artifact” used by the students, but also plays the role of “subject” and “community” and changes the “rules” and the “division of labor” (Souto and Borba 2016), contributing to the production of meaning.

In other words, in a collective of humans-with-media, human and non-human actors both have agency (Borba 2012), because non-human actors are not just tools that humans use (de Freitas et al. 2017). They act and interact together, distributing the nature of the agency between the actors, producing a reorganization of thinking that contributes to a production of knowledge. Humans are shaped by technology and technology is shaped by humanity (Borba 2012). In video production with mathematical content, members of such a collective work together to produce meaning related to mathematical learning in a video format. Social semiotics assists in the analysis of this meaning, taking into account not only the video produced, but also its production process and the negotiations among actors in a collective of humans-with-media. This analysis of the context can help in the discussion of the production of the knowledge by the sign-maker.

We can call the sign produced by the sign-maker a sign of learning (Bezemer and Kress 2016; Kress 2010). According to Bezemer and Kress (2016):

All signs are signs of knowing and learning, regardless of the mode in which they are made. The affordances of each mode entail that the sign-makers learn and demonstrate differently. The wider the range of resources made available to learners in environments of learning, the wider the range of 'evidence' available to the learner, and the wider the range of their inroads into learning (p. 50).

In the framework of social semiotics, learning occurs through engagement with the world. In traditional frames "[…] learning is what happens in schools of some kind, with teachers and a curriculum. In our frame, learning is the inevitable outcome of any and every engagement with the (socially made) world." (Bezemer and Kress 2016, p. 37).

In addition to the theories of social semiotics and multimodality, we adopt Borba and Villarreal (2005) idea that learning is collective, permeated by non-human actors (in the case of this research, video media production) and human relations. All of these actors influence the focus of the individual, because what happens on TV, what happens on social networks, what happens at home before going to mathematics class, and the opinions of colleagues, for example, all contribute to the reorganization of the individual's thinking. In this way, we can look at individuals and seek indications of their learning through their interaction with human and non-human actors, since, in the same classroom, in the same activity, different people will be learning different issues at different rates.

In the next section, we explain the research methodology. Then we explore the work of collectives of humans-with-media during the production of videos with mathematical content, based on a social semiotics analysis.

3 Methodology: video production in the classroom

The research was conducted with middle school students in three municipal schools situated in the city of Blumenau (in the state of Santa Catarina, in the South of Brazil): EBM[1] Felipe Schmidt, EBM Quintino Bocaiúva and EBM Wilhelm Theodor Schürmann. In municipal schools, students range in age from 6 to 14. Students aged 13–14 were chosen to participate in the research, because their teachers and the researchers believed they could explore more mathematics content because they had already studied various content previously. In two schools, students were able to choose the mathematical content to explore in their videos. In one of the schools, the teacher asked them to create a video with the content of functions, which was a theme they were studying in that quarter.

In each school, students were divided into groups (they chose the group members according to their existing friendships). Each group produced a video with mathematical content using their chosen approach (including video lesson, animation, role play, and so on). In all, 19 videos were produced.

To create data, we looked for a methodology for the production of classroom videos, but we did not find an extant systematization for data production. As a result of our experience, we believe that classroom video production can be divided into six steps: (i) Conversation with students and presentation of video types; (ii) choice of a theme and research about the topic; (iii) formulation of the script; (iv) Video recording; (v) video editing; and (vi) dissemination of the videos (Oechsler and Borba 2017; Oechsler et al. 2017). In all schools, the steps involved in the video production were the same, other than the school in which the teacher pre-selected functions as the topic. In this school, the steps were followed as described, including researching the topic, except that the topic had been chosen in advance. Each of these steps was part of the data collection for this research. Each step was recorded in audio and video. These recordings, along with the field diary, interviews, and the videos produced, constituted the research data. The researcher followed all the steps in the classroom, along with the teacher. At times, the researcher was an observer of the negotiations between the students. At other times, researchers were asked to participate in the discussions themselves and ask questions about mathematical content or video recording techniques. This aspect characterizes the methodology as action research.

The production of these data used the triangulation technique, which involves using multiple different procedures

[1] Acronym meaning Municipal Elementary School.

Fig. 1 Conversation between the researcher and the students during the class when they were producing the script. Source: survey data

Researcher: $\frac{2}{5}+\frac{3}{7}$, *how would you do it?*
Student 1: I would do 2 + 3 and 5 + 7.
(...)
Researcher: Ok. Look, what are you guys going to do? What are you going to work on as a fraction?
Student 1: We are going to work on the fraction and the number.
Researcher: Ok. And what does 3/5 mean? [conversation between the researcher and the students that produced the "Sum of Fractions" video. We can see that, when summing the fractions, the student adds the numerators and the denominators, which is incorrect. To highlight this, the researcher asks the student questions about the meaning of the fraction.]

for producing data. According to Lincoln and Guba (1985), this technique helps in the interpretation of the data, thus leading to greater credibility when data are analyzed. This is a qualitative methodology, in which the main focus of analysis is the process involved in the production of the videos, highlighting the choices made by the producers of the materials (Oechsler and Borba 2017). As stated above, under a social semiotics framework, it is not enough just to analyze the final version of a video—it is necessary to follow the production process in order to understand the discussions and negotiations of the producers throughout. By analyzing the process, it is possible to notice that some discussions and negotiations are repeated, allowing one to infer that the data observed in this research would also be observed in other video production processes.

In the next section, we explore and analyze the process of the video production, based on social semiotics and the views we present above regarding media and mathematics education. As a case study, we chose the video "Sum of Fractions" (produced by a group of students at EBM Quintino Bocaiúva) because it highlights important elements for the discussion of how the production of videos in the classroom can help in the communication of mathematical knowledge and in changing the dynamics of the classroom. Analyses of other videos produced in the research can be found in publications by Oechsler and Borba (2017, 2018).

4 Video analysis under social semiotics

In this section we look at the video production process, taking the video "sum of fractions" as an example. However, it should be noted that discussions and negotiations similar to those that took place in the group that made "sum of fractions" were also observed in other groups, allowing us to find common characteristics across different groups.

Before starting the video production process, students had to choose the mathematical content that they wanted to explore, and decide how they would explore it in the video. In the "sum of fractions" video, students opted to explore the sum of fractions because they believed it would be an easy topic to explain in a video. The students in question were 13–14 years old, and the first time that they had had contact with fractions content was at 9–10 years old, which led them to believe that it would be an easy topic to explain. Other groups opted for other content, such as equations, potentiation and percentage. What we observed was that, generally, the choice was based on the affinity they thought they had with the content—students chose topics that they thought would be easiest to explain.

At the second stage of video production, students needed to 'research' the subject. A very important ally in this process was the Internet. Students used this non-human actor to search for content, finding definitions, examples, and applications. When bringing this research to the classroom, they confronted their knowledge and discussed ideas with both the class teacher and the researcher. This was an opportunity for the adults to perceive the students' difficulties directly.

In the case of this video, they had chosen fractions as their topic, and needed to study it in order to delimit what to explore in the video. Figure 1 presents a conversation that took place in the classroom between the researcher and the students during the development of the project.

After this discussion, students noticed their difficulties in the content and got back to the 'research'. With the help of the researcher (a human actor), as well as Internet search and textbooks (non-human actors), they observed that they were mistaken about the procedure for summing fractions. After this, they decided that they would explore the fraction sum algorithm because, as they had done, several students mistakenly used the procedure of summing numerators and denominators.

Other groups also came to realize their difficulties when choosing the theme of the video. Some chose to explore potentiation, but did not know how to explain why any number raised to zero, except zero, results in the number one. Problems with properties of square rooting and solving equations were also found in other videos. In these discussions between students and between students and teacher, it was evident that many students chose content that they initially thought was easy, but when exploring their definitions and properties, difficulties appeared, as in the case of

Table 1 Image from the video of the EBM Quintino Bocaiúva group computing the Sum of Fractions. Source: survey data

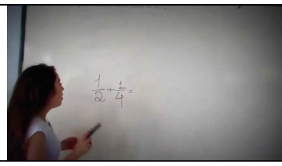
	*Now let's add the fractions. Is equal to...*$\frac{2}{6}$*. But is it right?*

Fig. 2 Images of the video of the EBM Quintino Bocaiúva group as the student was solving the sum of fractions. Source: survey data

the sum of fractions with different denominators, or when solving a first-degree equation, where students insisted that unknowns needed to be on the first limb, or that numbers ‘pass the other side of equality’ with the ‘opposite sign’, regardless of additive and multiplicative properties.

Difficulties like these, involving core concepts relating to the content, would not be perceived by the teacher in an examination, because students would not verbally express their actions in solving the problem. They would solve the equation or operation presented to them and would often get the result right, but they would not know why to perform a particular operation. By externalizing their actions in the video, the students’ difficulties become visible, allowing the teacher to explore them during the process. This is one of the advantages of following the production process, an aspect supported by social semiotics, since, in negotiations and discussions among students, it is possible to realize their difficulties and explore them; it is possible to understand the social process that generated the video (for example).

After the research and discussion stage, students began to negotiate how to explore their chosen content in the video. Will we do a video lesson? An animation? Will we use any software? In the case of the video “sum of fractions”, they decided to make a video lesson, using a picture and fraction disks. The video featured an introduction with a student explaining the parts that make up the fraction (numerator and denominator) and the sum of two fractions. In this explanation, students decided to explore the error itself, adding the numerators and denominators of the fractions together, as we can see in Table 1.

The student did the operation, pointing to what she was doing, and, when asking about the veracity of the answer, looked at the camera and made a facial expression of doubt, along with a gesture of opening the arm with the palm of the hand facing up, which also characterizes doubt (Fig. 2). In this section of the video, we observe the use of different modes, such as mathematical symbolic language (in the writing of the fractions), moving image, gestures, facial expression, and orality. The combination of these modes was intended to explain the fractional sum procedure and lead the viewer to reflect on the action taken, by asking if what was displayed was right. In a written presentation of this problem, students are expected to present the correct answer, without making the error. With video, on the other hand, they can combine different modes (writing, speaking, gestures and facial expression) to highlight a common problem in the solution, leading the viewer to reflect on it. They can exploit this error by arguing that the sum of fractions must be performed from equivalent fractions. Thus, they explore not only the operation/calculation, but also more general concepts of fractions, reorganizing their thinking and expanding the production of knowledge.

After questioning the initial, incorrect answer, students used fraction discs to explore the equivalent fraction content and solve the proposed problem. Fraction discs were chosen by the students to visually illustrate the concept of equivalent fractions, which plays a role in the algorithm used in the addition and subtraction of fractions with different denominators. The sign-makers knew that other students had difficulties with this subject, just as they had, and they decided that a visual representation, coupled with an oral explanation, could help other students understand the problem (Table 2). Typically, this exploration is accomplished through symbolic mathematics in the classroom, and the fractions are converted to the lowest common multiple to solve the problem. However, doing these same operations in the video was not considered by the students. In their view, this method did not help them to understand the sum of fractions. For this reason, they chose to use modes other than symbolic language to explore the concept of equivalent fractions and solve the problem. According to them, this

Table 2 Images of the video of the EBM Quintino Bocaiúva group using fraction discs to find fractions equivalent to given fractions. Source: survey data

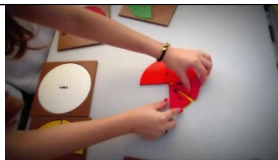
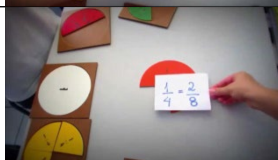
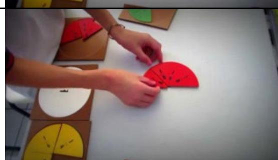
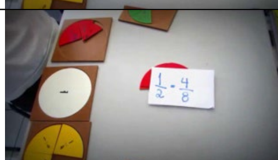
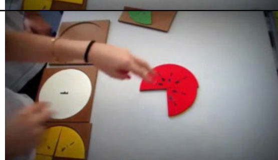
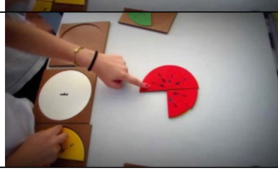
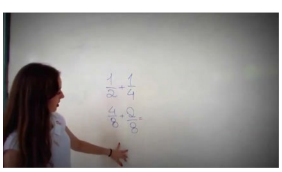
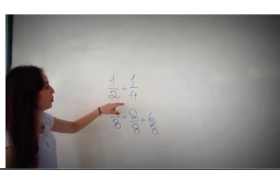
	*Of these parts, how many represent?* $\frac{1}{8}$*? No. Let's add another.*
	*Then.*
	*And for ? Let's think the same way.* $\frac{1}{8}$ *plus* $\frac{1}{8}$*, plus* $\frac{1}{8}$ *and plus* $\frac{1}{8}$*. Equal to* $\frac{4}{8}$.
	*Then,* $\frac{1}{2} = \frac{4}{8}$.
	*Now that we have fractions with the same denominator, let's add up.*
	*Then let´s add.* $\frac{2}{8}$ *plus* $\frac{4}{8}$ *equals 1, 2, 3, 4, 5, 6 eighths.*

Table 3 Images of the video of the group of EBM Quintino Bocaiúva effecting the Sum of Fractions with equivalent fractions. Source: survey data

	*So let's add.* $\frac{1}{2}$ *is equal to* $\frac{4}{8}$*. Adding with* $\frac{1}{4}$ *, that is equal to* $\frac{2}{8}$.
	*Adding these two fractions ... The result is* $\frac{6}{8}$*. Then* $\frac{1}{2} + \frac{1}{4}$ *is equal to* $\frac{6}{8}$.

helped them to understand the calculation and could help the viewers as well.

As shown in Table 2, the student used fraction discs as a visual representation of the fractions explored in the video; mathematical symbolism in the exposition of the equivalent fractions ($\frac{1}{4} = \frac{2}{8}$ and $\frac{1}{2} = \frac{4}{8}$); and spoken language explaining exactly what is displayed in the moving image and the gesture, emphasizing the explanation. After this action, the students returned the image to the table and

Fig. 3 Interview with the students about what they learned producing the video. Source: survey data

Researcher: What did you learn [with the video production]?
Student 2: For me, [my knowledge] has improved a lot, because I understood almost nothing about fractions.
Researcher: Ok. Did you learn something?
Student 2: Yes.
Student 1: So, I learned a little too. So, we remembered, because we had already studied fractions. Then some of us still remembered a little of what we had studied before. It helped a lot.
[conversation between the researcher and the students that produced "Sum of Fractions" video about what they learned during the activity.]

Source: survey data

the student used the equivalent fractions found with the aid of the manipulative material to solve the problem of the sum of fractions on the whiteboard (Table 3).

Again, the student made use of mathematical symbols (fraction, plus, and equality) to show her computation; orality, to explain what she was writing in the form of symbols; and gestures, to emphasize what was said orally in the explanation, trying to show the viewer what was done in the resolution of the operation.

In all of the scenes of the video, we can see the use of several modes such as writing, symbolism, orality, gestures and facial expression to explain and emphasize the sum operation, used in a combined way, characterizing multimodality. Orality, connected with gestures, helps the viewer understand what the student was writing (in the symbolic mode) in the board. Facial expressions emphasize that there was a problem with the initial solution. Together, these modes draw the attention of the viewer to the mistake that was made (intentionally) in the solution. In the video sequence, didactic material allows for visualization of the equivalent fractions, emphasizing that, when adding fractions with different denominators, it is necessary to use equivalent fractions. In this example, we can see that the sign-makers explored the affordances of the mode to produce meaning. If they opted for another type of video, using slides or even just writing on paper (as we do when we solve a test), the mathematical communication would be modified, potentially increasing or reducing the options for explaining the content and thereby altering the final result. Here, due to the affordances of the video modality, we see a mix between mathematical language (O'Halloran 2000, 2005) and cinematographic language. Each mode was used to achieve a common goal: to communicate the mathematical content. The combined use of these modes multiplies their potentialities (Lemke 1998), aiding in the production of mathematical knowledge. The affordances of the technology have agency, as the plasticity of video, and its 'multimodal character' allow students' ideas to be communicated in a dynamic, multimodal way. This implies the reorganization of thinking (Borba and Villarreal 2005; Borba et al. 2018).

The monitoring and analysis of the video production process allows the teacher/researcher to understand the reflections and negotiations of the students during the process. The emphasis given to the process of production by social semiotics helps the teacher understand students' doubts and to identify the reorganization of thought for the production of meaning (Fig. 3).

It should be noted that student 1, who, in the excerpt above, says that she learned a little, did not know how to compute the sum of fractions at the beginning of the video production process. When asked about the sum $\frac{2}{5} + \frac{3}{7}$, she answered $\frac{5}{12}$. Then, in the video, it was this student who explored the fraction sum in the frame. Bezemer and Kress (2016) argue that communication and learning are interconnected, since the individual communicates what has made sense to him or her. This allows us to identify signals of the individual's learning of the communicated content. The student's exploration of the content with which she previously had difficulty indicates a learning signal (Bezemer and Kress 2016; Kress 2010) in communicating the mathematical content. The observation process allowed us to understand the reorganization of mathematical thinking in the interaction of producers with a collective of human actors (colleagues, teacher, researcher, etc.) and non-human actors (e.g., Internet, books, fraction discs and video itself). It also allowed us to understand the choice of the modes, and how video signals the learning of these students (Bezemer and Kress 2016; Kress 2010), who, in producing it, came to understand the procedure of adding fractions.

Kress (2010) explores the meaning of a particular theme for an individual by describing the activity of producing a map by visitors to a museum in London. Semiotically and communicatively speaking, maps are the answer to a request. Pedagogically speaking, maps are signs of learning (Kress 2010). We can draw parallels between the work of Kress (2010) and the analysis of videos produced by the students in our study. Semiotically and communicationally speaking, videos were the response to a request made in the classroom. Pedagogically speaking, videos reflect students' learning signals: what they understood about the content and what they are communicating.

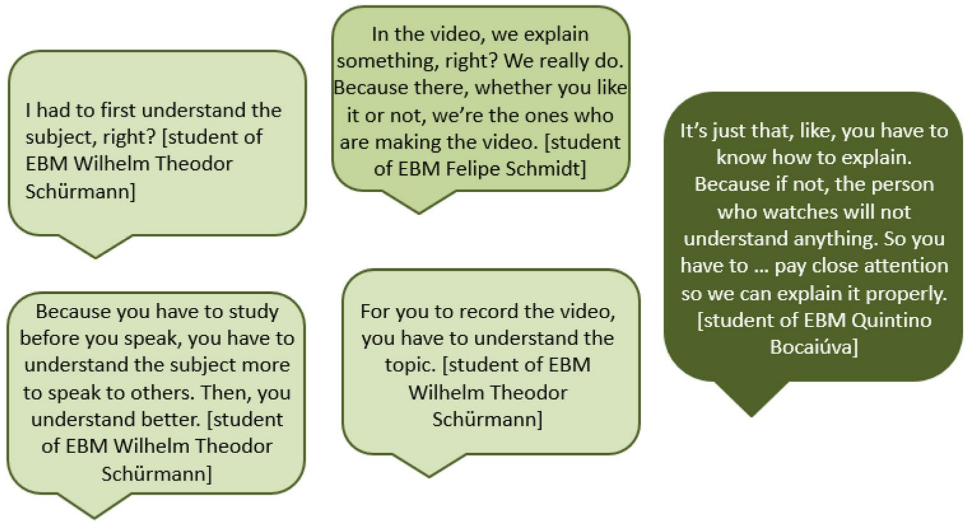


Fig. 4 Students' talk about video content reporting. Source: survey data

At the end of the research, we conducted interviews with each of the video producer groups. Our intention was to identify what the students learned from the activity. In Fig. 4, we present some students' comments, recorded during the research, which expose their conceptions about what was reported in their videos (Fig. 4).

As noted in the comments from the group that made the video "Sum of Fractions" (Fig. 3), other student groups emphasized that, to explain the subject in the video, they had to study and understand the content. They could not just talk about content—first, they needed to understand what to communicate. This can be construed as a sign of learning (Bezemer and Kress 2016; Kress 2010): the video produced is the result of students' learning about the content being explored. The students who created the video "sum of fractions" also highlighted the need to study in order to explain the subject in the video.

At the end of the process, when all the videos had been produced, they were presented to the rest of the class, which generated discussions about the production. After watching the "sum of fractions" video, one student from the class indicated that he did not understand why $\frac{1}{4} = \frac{2}{8}$. Another student in the class, whose group had produced a video on fraction classification, explained that these are equivalent fractions, as he had explained in his own video. The process of producing a video on the subject helped this student to understand equivalent fractions (Oechsler and Borba 2017). In addition, the discussion during the screening allowed this concept to be explored with the class and not just with the groups that used it in the videos.

In fact, the videos can even go beyond the original classroom: once produced, they can be posted on video hosting platforms and other social media. These and other videos can then be viewed by other students, and used by other teachers in their own classrooms. This represents a change in the dynamic of the classroom: the teacher is no longer the sole holder of knowledge. Students can now express their mathematics ideas through videos and find other videos to help them understand the content. The teacher serves as a mediator, assisting the student in the search for learning, reflecting on the communication in the video and searching sites for the improvement of ideas. In this way, video production creates a dialogue between teacher and student in the search for knowledge. Teacher, students, media, video production equipment, Internet, books, and so many other actors, in a humans-with-media construct, act together in the production of knowledge (Borba and Villarreal 2005). To produce mathematical videos, students need to understand the content deeply. Otherwise, they could make a conceptual or operational error. This motivated them, as seen in the excerpts in Fig. 4, to study and really understand what they had chosen to explain, producing knowledge.

5 Conclusions

More than a decade ago, Borba (2009) speculated that with the development of software and hardware, most problems that are usually addressed in the classroom could easily be solved if the Internet were allowed in the classroom. As a matter of fact, if the Internet is allowed in the classroom, and the collective of humans-with-media includes the Internet, many types of traditional problems can no longer be considered 'problems' at all. In other words, a problem also depends on what media (paper and pencil, Internet, etc.) are allowed in the educational process of solving it. At the time of Borba (1999) writing, one-response textbook questions and exercise lists could be trivially solved by WolframAlpha, and these days they can be solved by software such as Photomath. Borba predicted that digital mathematical performance (Scucuglia 2015) and modeling activities would survive, while these trivial exercises would fade away. Over the years, it has grown more and more clear that the classroom can no longer fit into four walls, one ceiling and one floor. The Internet, and especially the mobile Internet, has changed this geometry. Videos produced by collectives of humans-with-digital-technologies is produced inside and outside the traditional classroom, and when it is posted on video sharing sites like YouTube, it is also presented inside and outside the classroom.

In the research reported in this paper, we observe a distinct change in the dynamics of the classroom through the production of videos. The student is not just a passive receiver of information, or, to use Freire (1970) analogy, the student is not a bank account to receive deposits. Instead, the student can become a protagonist, able to decide how the content will be explored and how it will be shown to colleagues. It should be noted that, throughout the activity, students were always concerned with producing knowledge through their videos. For them, it was always necessary for the viewer to understand what was being explored. For example, in the video we discussed in this paper, the producers themselves, prior to the production of the video, had difficulty with the concept of adding fractions. They argued that, like their experience, there were several students who had the same difficulty—so why not explore this difficulty in the video, initially showing the wrong solution and then, from there, discussing the correct way to solve the sum of fractions? They opted for that approach so that other students could understand the subject the way they did, perceiving the error, and analyzing the use of equivalent fractions. We can thus see video as a teaching tool, with students teaching viewers through knowledge production, discussing and reflecting on the error, and searching for a solution in different modes.

In order to produce the video successfully, the sign-makers needed to interact with various actors who helped them reflect on the operation in question. This interaction of students with other humans and media allowed them to reorganize their thinking according to the affordances of the media (Souto and Borba 2016). In the video explored in this paper, the students interacted with each other and with the researcher and the teacher (human actors), as well as with the Internet, textbook, camera, fractional didactic material, and editing software (non-human actors). Through these interactions, students reflected on the content to be explored (summing fractions) and opted to produce a video-lesson that mixed image and didactic material. The completed video, which was created as a result of collectives of humans-with-media, then becomes another member of the collective, with its own agency.

The agency of sign-makers and the actors they interact with can influence the meaning that they produce. It is thus important to be aware of all of the negotiations, reflections and discussions of the sign-makers in order to understand their choices and how these can influence the meaning and knowledge produced (Bezemer and Kress 2016; Kress 2010). With this activity, we can see that video changes how mathematical content is communicated, because the sign-makers can use other modes than just mathematical language to explore and explain what they are doing. This allows a reorganization of the sign-makers' thinking, exploring the affordances of the media.

This reorganization of thinking can lead to the production of knowledge (Borba 2012). Such reorganization can be seen in a new light if we consider the work of Kress (2010), who indicated that the production of a map by students is a sign of their learning. In much the same way, we have seen that the videos produced by students are signs of their learning: through video production, they show their understanding of the content. In other words, video is the meaning and the knowledge produced by sign-makers as they interact in a collective of humans-with-media. Through this activity, the students themselves became aware of their difficulty and sought ways to overcome it. In this way, video production can be considered a teaching and learning tool, through encouraging students' discussion and reflection about content and its exposition so as to produce meaning. After all, one can communicate only what knowledge one has produced.

In addition to allowing differentiated communication of mathematical knowledge, by encouraging the use of modes beyond mathematical language, the production of videos promotes a change in the dynamics of the classroom, and contributes to breaking the barrier between the classroom and the outside world. The videos that were produced can be released outside the classroom (on social media and video hosting platforms, at festivals, and so on) and assist other students, teachers, and members of society in general in understanding a given subject. Just as videos available in the media can enter the classroom through the questions of a student (as shown at the beginning of the paper), videos produced in the classroom can spread beyond the classroom when they are posted in the media for any user to access. In this sense, videos, posted on social networks 'break the walls' of the ordinary classroom.

Let us return to the example we gave at the beginning of the paper, about the student who searched for an Internet video to study the content explored by the teacher on the day he missed the lesson. Later that semester, the teacher requested that the students produce a video about probability and/or combinatorial analysis. Connecting with his previous question, this student and his group chose to produce a video about the probability of winning a game of the Mega Sena,[2] a Brazilian lottery, demonstrating the students' learning about the content that was found on the Internet and discussed in the classroom. This episode reinforces the presence of the Internet and other media in the classroom, reorganizing the school space: from a question generated by an Internet video (about content previously explored in the classroom), discussions were carried out and, through video production, new signs of learning emerged. This video was later sent to the Festival of Digital Videos and Mathematics Education in 2018, which allowed the student's learning to propagate outside the school space and reach other environments. No doubt this video could also generate questions in another viewer, who could then take it to another classroom and generate new discussions, reflections, negotiations and new videos. Here, the Internet is serving as an ally of education as it breaks the boundaries of the classroom. It promotes the reorganization of thinking through a collective of humans-with-media, involving both human and non-human actors. This interaction allows the production of mathematical and technological knowledge, as seen in the videos produced. A video is a product of collective of humans-with-media and may become part of a new collective. This is a new frontier for education. How can we keep up with it, and how can we go further, fostering more change that brings the mathematics classroom closer to the rest of our lives?

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[2] The Mega Sena is a Brazilian lottery game in which the bettor must choose 6 numbers out of 60. The bettor who hits the six numbers drawn wins the prizes.

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